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Decomposing swap spreads ☆

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Abstract

We analyze a six-factor model for Treasury bonds, corporate bonds, and swap rates and decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit risk element from the underlying LIBOR rate, and a factor specific to the swap market. The convenience yield is by far the largest component of spreads. There is a discernible contribution from credit risk as well as from a swap-specific factor with higher variability which in certain periods is related to hedging activity in the mortgage-backed security market. The model also sheds light on the relation between AA hazard rates and the spread between LIBOR rates and General Collateral repo rates and on the level of the riskless rate compared to swap and Treasury rates.

JEL classification: C51; G12; G13

Keywords: Swap rates; Term structure; Credit risk; Liquidity

1. Introduction

Interest rate swaps and Treasury securities are the primary instruments for hedging interest rate risk in the mortgage-backed security (MBS) and corporate bond markets, but the large widening of swap spreads—the difference between swap rates and comparable Treasury yields—in the fall of 1998 clearly revealed that there are important differences between the Treasury and swap markets. The ability to accurately hedge interest

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rate risk critically depends on understanding these differences. This paper decomposes the term structure of swap spreads into three components: a convenience yield for holding Treasury securities; a credit spread arising from the credit risk element in LIBOR rates, which define the floating-rate payments of interest rate swaps; and a residual component. As we will explain below, the size of the convenience yield is what separates the Treasury yield from the riskless rate and it is by far the largest contributing factor to the swap spread. The two other components separate the swap rate from the riskless rate. The credit risk component does not contribute much to the time variation of spreads. Starting with the onset of an MBS refinancing period towards the end of 2000, the swap factor pushes the swap spread down influenced by, among other things, MBS hedging activity. The dynamic decomposition of the evolution for the 10-year swap spread is depicted in Fig. 1.

We obtain these decompositions through a joint pricing model for Treasury securities, corporate bonds, and swap rates using six latent factors. Two factors are used in the model of the government yield curve, one factor is used in modeling the convenience yield in Treasuries, two factors are used in the credit risk component in corporate bonds, and one is a factor unique to the swap market.

We follow Collin-Dufresne and Solnik (2001) and find the fair swap rate by pricing the cash flows of the swap separately using an estimated riskless rate. This is reasonable given the fact that counterparty risk on a plain vanilla interest rate swap is typically eliminated by posting collateral and netting agreements. Furthermore, Duffie and Huang (1996) demonstrate that the effect of counterparty credit risk on the fair swap rate in interest rate swaps is extremely small.

The role of credit risk in our modeling of swap spreads is therefore related to the level of the LIBOR rate used for defining the floating-rate payment on the swap. As observed in Sun, Sundaresan, and Wang (1993) and Collin-Dufresne and Solnik (2001), it is the refreshed credit quality of this rate which causes swap rates and AA-rated corporate curves to be different. A main focus of our paper is how the credit risk component in LIBOR rates contributes to the swap spread. If the credit risk element of LIBOR is expected to increase over

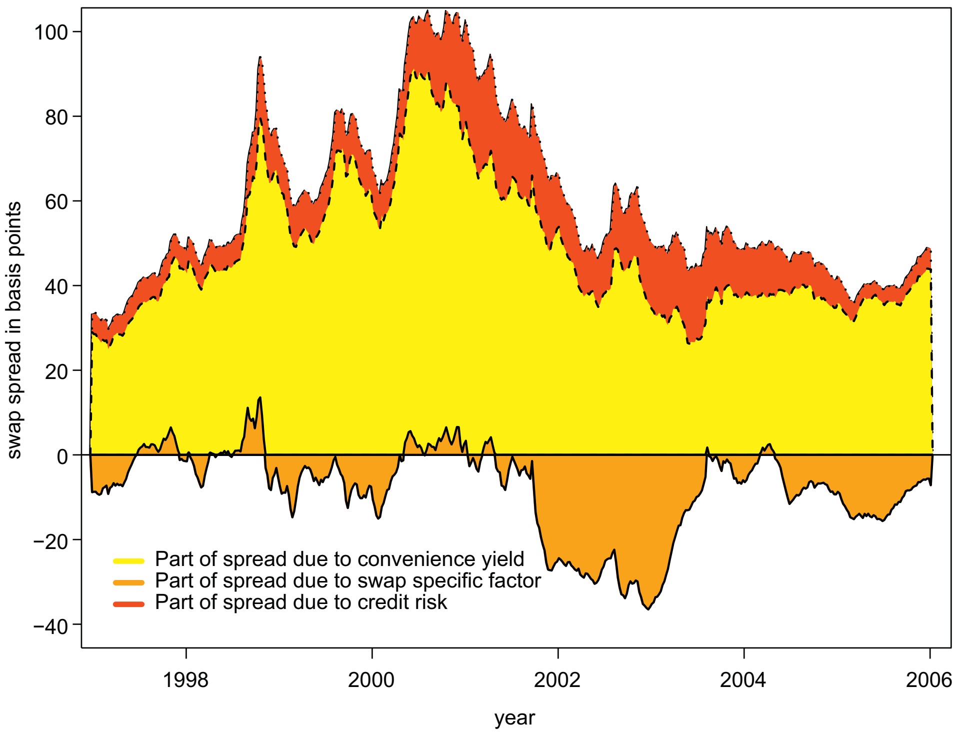

Fig. 1. Decomposition of the estimated ten-year swap spread into a swap-specific factor, LIBOR credit risk, and a Treasury convenience yield. The estimation is based on weekly U.S. swap rates, Treasury rates, and corporate bond yields for banks and financial firms for the period December 20, 1996 to December 30, 2005. The factors are estimated using the Kalman filter. On dates where the effect of the swap-specific factor is negative, the convenience yield is the sum of the light-shaded and medium-shaded areas on the graph.

time under the pricing measure, this will imply a larger credit-related contribution to swap spreads in the long end of the curve—a point also made in Collin-Dufresne and Solnik (2001).

To obtain evidence on the possible future paths of LIBOR, we look at corporate bond data from the banking and financial sectors. Clearly, if AA credit quality is expected to decline, this will affect the long end of the AA curve. But the long end of the AA curve also reflects the possible risk of migration into lower credit rating categories. We therefore set up a rating-based model for corporate curves to consistently separate the part of the AA corporate spread that is due to changes in the risk of defaulting directly from AA from the part that is due to the risk of a credit downgrade. In addition to ensuring consistent modeling of the AA corporate curve, the rating-based modeling has other benefits. The corporate bond yield curves for the different categories are noisy, but the inclusion of several curves makes our model less sensitive to measurement errors in these curves. Also, as shown empirically in Lang, Litzenberger, and Liu (1998), there is a positive relation between swap spreads and A spreads. Through our modeling this relation is quantifiable.

Our model allows us to check the assumption of homogeneous credit quality in the sense defined in Duffie and Singleton (1997). There—and in Collin-Dufresne and Solnik (2001)—it is assumed that the 6-month AA corporate rate and 6-month LIBOR are the same. We use AA banking rates as the corporate AA curve and find that a model in which LIBOR is set based on AA corporate curves cannot fit the swap curve completely, although on average the assumption is not unreasonable. We show that in the latter part of the sample, hedging activity related to the mortgage market is a very likely explanation for the deviations in swap spreads from the levels that would hold under a homogeneous credit quality assumption.

The largest component of the swap spread, however, is a convenience yield for owning Treasuries. We find that this convenience yield is plausible when compared to evidence from the agency bond market. We are then able to assess the validity of the approach of Grinblatt (2001), who views swap rates as riskless rates and the spread between government and swap rates as a liquidity spread. His argument is that AA refreshed credit is virtually riskless. While it is true that the historical default experience for AA issuers over a 3-month or 6-month period is extremely low, we do find a credit risk component in swap spreads and also that the swap-specific factor can drive the swap rate away from the riskless rate.

Our model builds upon and extends a number of previous models and empirical studies. In Duffie and Singleton (1997) the 6-month LIBOR rate is based on an adjusted short-rate process R which includes the Treasury rate, an adjustment for liquidity differences in Treasury and swap markets, and a loss-adjusted default rate. By simultaneously using R to discount the cash flows of the swap to determine the floating-rate payments of the swap, the fair swap rates depend only on R and not on the contributions from the individual components to R. In their subsequent analysis, the swap rates are therefore regressed on proxies for liquidity and credit risk, but the components are not included separately in the pricing model.

Our approach is similar to that of Liu, Longstaff, and Mandell (2006), who use a five-factor model with three factors to model Treasury yields, one factor to model the 'liquidity' (i.e., what we refer to as the convenience yield) of Treasury securities and one factor for default risk. Their identification of the credit risk factor and the liquidity component in swap spreads relies critically on the use of 3-month General Collateral (GC) repo rates as a short-term riskless rate and 3-month LIBOR as a credit-risky rate. Their default factor is in fact equal by definition to the difference between 3-month LIBOR and 3-month GC repo rates, an assumption used also (for 1-month rates) by He (2001) and Li (2004). By including information on corporate bonds in our study we do not need to rely on short-term interest rate spreads as proxies for credit risk and Treasury components, and this strongly alters conclusions about the size and time-series behavior of these components.

Most of the various proxies that we discuss in this paper can be found in the model of Reinhart and Sack (2002), who specify a multivariate time-series model for 10-year swap rates, off- and on-the-run Treasury rates, Refcorp rates (to be defined below), and AA corporate rates. They identify a swap-specific factor, but their model is not a full pricing model and therefore it does not allow one to quantify the term structure effects of this factor.

The rating-based approach explicitly incorporates different dynamics for bonds of different rating categories. This is consistent with empirical evidence in Duffee (1999) who finds that the dynamics (and not just the levels) of the hazard rate process depend on the rating category. We incorporate this finding into our model by letting the default intensity process evolve as a diffusion with regime-shifts, i.e., a diffusion with rating-dependent parameters. An alternative approach would be to model the default intensities for different rating categories by adding positive-valued processes for lower categories, but unless we include a migration component as well we cannot price bonds consistently.

The outline of the paper is as follows. In Section 2 we describe the structure of our model. The explicit pricing formulas are relegated to an Appendix. Section 3 describes the U.S. market data that we use, and Section 4 explains our estimation methodology. In Section 5 we report our parameter estimates along with residuals from the estimation, and we elaborate on our main findings. Section 6 concludes.

2. The model

Our model of Treasury bonds is an affine short-rate model with a liquidity component, and we use an intensity-based, affine framework for corporate bonds and swaps as introduced in Duffie and Singleton (1997, 1999) and Lando (1994, 1998). Since our pricing of corporate bonds includes rating information we also use the affine, rating-based setting introduced in Lando (1994, 1998).

We use a six-factor model based on independent translated CIR processes. More precisely, we assume that the latent state vector X consists of six independent diffusion processes with an affine drift and volatility structure,

$$X_t = (X_{1t}, \ldots, X_{6t})',$$

$$\mathrm{d}X_{it} = k_i(X_{it} - \theta_i)\,\mathrm{d}t + \sqrt{\alpha_i + \beta_i X_{it}}\,\mathrm{d}W_i^P, \quad i = 1, \ldots, 6,$$

where the Brownian motions $W_1^P, \ldots, W_6^P$ are independent. This specification nests the Vasicek ($\beta = 0$) and CIR ($\alpha = 0$) processes as special cases. We assume that the market price of risk for factor i is proportional to its standard deviation and normalize the mean of X_i under Q to zero for identification purposes, so the processes under Q are given by

$$\mathrm{d}X_{it} = k_i^* X_{it}\,\mathrm{d}t + \sqrt{\alpha_i + \beta_i X_{it}}\,\mathrm{d}W_i^Q,$$

where

$$k_i^* = k_i - \lambda_i \beta_i,$$

$$\lambda_i = -\frac{k_i \theta_i}{\alpha_i}.$$

A multifactor affine model in which the factors are independent is restrictive as pointed out by Dai and Singleton (2000), among others. The advantage is that pricing formulas have explicit solutions, and the model is more parsimonious with fewer parameters to estimate. Since the empirical section shows that average pricing errors in the model are close to zero and the standard deviations of pricing errors are comparable to those of other studies, the independence assumption is not likely to be a serious component of model misspecification.

From the state vector we now define the short-rate processes, intensities, and liquidity adjustments needed to jointly price the Treasury and corporate bonds and the swap contracts.

We work in an arbitrage-free model with a riskless short rate r given as a three-factor process,

$$r(X) = a + X_1 + X_2 + (e + X_5), \tag{1}$$

where the first two factors X_1 and X_2 are the factors governing the Treasury short rate while the last factor X_5 is a Treasury premium which distinguishes the Treasury rate from the riskless rate. The premium is a convenience yield on holding Treasury securities arising from, among other things, (a) repo specialness due to the ability to borrow money at less than the GC repo rates,[1] (b) that Treasuries are an important instrument for hedging interest rate risk, (c) that Treasury securities must be purchased by financial institutions to fulfill regulatory requirements, (d) that the amount of capital required to be held by a bank is significantly smaller to

[1]See Duffie (1996), Jordan and Jordan (1997), Krishnamurthy (2002), Cherian, Jacquier, and Jarrow (2004), and Graveline and McBrady (2006).

support an investment in Treasury securities relative to other securities with negligible default risk, and to a lesser extent (e) the ability to absorb a larger number of transactions without dramatically affecting the price. The constant a is the Q-mean of the government short rate while e is the Q-mean of the convenience yield. Consequently, the Treasury short-rate process is given as

$$r^{g}(X) = a + X_1 + X_2. \tag{2}$$

The Treasury premium is positive and in the empirical work we restrict the parameters such that $e + X_5$ is positive. From this affine specification, prices of zero coupon government bonds are given as

$$P^{g}(t, T) = \exp(A^{g}(T - t) + \mathbf{B}^{g}(\mathbf{T} - \mathbf{t})' X_t),$$

where A^{g} and $\mathbf{B}^{g}$ can be found in Appendix B, which also give a formula for the par rates used in the empirical work.

Our model for corporate bonds prices a 'generic' bond with initial rating i by taking into account both the intensity of default for that rating category and the risk of migration to lower categories with higher default intensities. Spread levels within each rating category are stochastic, but for all rating categories they are modeled jointly by a stochastic credit spread factor.

We use the reduced-form representation with fractional recovery of market value to price a corporate bond which at time t is in rating category $\eta_t = i$:

$$v^{i}(t, T) = E_t^{Q} \exp\left(-\int_t^T (r(X_u) + \lambda(X_u, \eta_u)\,\mathrm{d}u) \right), \tag{3}$$

where $\lambda(X, \eta)$ is the loss-adjusted default intensity when the rating class is η. The default intensities for the different categories are assumed to have a joint factor structure

$$\lambda(X, i) = v_i \mu(X),$$

where the v_i are constants, and $\mu(X)$ is a strictly positive process which ensures stochastic default intensities for each rating category and plays the role of a common factor for the different default intensities. We specify μ as

$$\mu(X) = b + X_3 + X_4 + c(X_1 + X_2).$$

Note that the process μ is allowed to depend on government rates through the constant c, while the two processes X_3 and X_4 are used only in the definition of μ. Hence we have in essence a two-factor model for credit spreads across the different rating categories. We now have the definition of the loss-adjusted default intensity as a function of the state variable process and the rating category, so all that is left to specify before (3) can be evaluated is the stochastic process for the rating migrations. We work with a 'conditional' Markov assumption as in Lando (1994, 1998) which means that the transition intensity from category i to category j is given as

$$a_{ij}(X_t) = \lambda_{ij} \mu(X_t), \tag{4}$$

where λ_{ij} is a constant for each pair $i \neq j$ and $\mu(X_t)$ is the same factor that governs credit spreads. This means that the intensities of rating and default activity are modulated by the process $\mu(X_s)$. As shown in Lando (1994, 1998), this specification generates closed-form pricing formulas for corporate bonds in all rating categories that are sums of affine functions. Lando also shows that it is possible to obtain affine solutions using a more general specification in which the eigenvectors of the randomly varying generator are constant but the eigenvalues are affine processes. Such a specification unfortunately makes it hard to control the positivity of the intensities and it does not allow us to model the individual transition intensities as separately specified affine functions of the state variables. If we used such a specification we would have to resort to solving systems of PDEs, as done for example in Huge and Lando (1999). For these reasons we have chosen the simpler specification.

Our specification of the intensities applies under the risk-neutral measure Q. This is all we need to understand the pricing impact of migration and default risk. Since it is not practically feasible to back out the risk-neutral intensities of default and migration from prices, we employ the same idea as Jarrow, Lando, and Turnbull (1997) and impose structure on the matrix of intensities under the risk-neutral measure by using the empirically observed average transition rates. If we made the extreme assumption that the average transition rates were the true rates, then the process μ would play the role of a randomly varying event risk premium. If, at the other extreme, we believed that the true default intensity at time t for class i were given as $v_i\mu(X_t)$, then the only risk adjustment would be through the risk adjustment of X—a situation related to diversifiable default risk in Jarrow, Lando, and Yu (2005) where the distinction between the two risk specifications is treated in more detail. Our specification is flexible enough to capture randomly varying transition and default rates, diversifiable default risk premia, and event risk premia. But we make no attempt at separating the contribution from these three sources to the risk-neutral intensities.

Following Lando (1994, 1998), the price of a zero-coupon corporate bond in rating class i at time t is of the form

$$v^i(t,T) = \sum_{j=1}^{K-1} c_{ij} E_t\left(\exp\left(\int_t^T d_j\mu(X_u) - r(X_u)\,\mathrm{d}u\right)\right), \tag{5}$$

where the constants c_{ij} and d_j are given in Appendix B. The specification implies that default probabilities, upgrade probabilities, and downgrade probabilities move in the same direction. Ideally, in times of high default probabilities the probability of a downgrade would increase and the probability of an upgrade would decrease. However, we use only investment grade corporate bonds in our empirical study and the main drivers of fluctuations of credit spreads in our model are the loss-adjusted default intensities and the downgrade intensities. A separate adjustment of upgrade intensities has very limited pricing effects and as mentioned above it would force us to price bonds numerically using a system of PDEs (a numerical study confirming this view is available on request).

Our model does not take into account the difference in tax treatment between corporate bonds and Treasury securities. Elton, Gruber, Agrawal, and Mann (2001) employ a 'marginal investor' tax rate argument and estimate the tax premium on corporate bonds to be significant. However, they measure bond spreads using Treasury bonds as a benchmark. The convenience yield that we estimate for Treasury bonds easily explains a spread of similar magnitude. Furthermore, evidence on Treasury security holdings seems to support the viewpoint in Grinblatt (2001) that tax-exempt investors such as pension funds, broker-dealers, or international investors would arbitrage away differences in yield due to a tax advantage. The fraction of Treasuries owned by investors for whom the tax advantage does not apply is indeed large: according to Fabozzi and Fleming (2005), as of September 30, 2003, foreign and international investors held 37% of publicly held Treasury debt, Federal Reserve Banks held 17%, pension funds held 9%, and state and local treasuries held 8%. Longstaff, Mithal, and Neis (2005) and Chen, Lesmond, and Wei (2007) find no or only weak support for a tax effect.

With the specification of the Treasury and corporate bond prices in place, we can now find swap rates. First, we need to define the 3-month LIBOR rate used to determine the floating-rate payment on the swap:

$$L(t,t+0.25) = \frac{360}{a(t,t+0.25)}\left[\frac{1}{v^{\mathrm{LIB}}(t,t+0.25)} - 1\right], \tag{6}$$

where $a(t,t+0.25)$ is the actual number of days between t and $t+0.25$ and $v^{\mathrm{LIB}}(t,t+0.25)$ is the present value of a 3-month loan in the interbank market. The adjusted short rate to value this loan is given as

$$\lambda^{\mathrm{LIB}}(X_s) = r(X_s) + v_{\mathrm{AA}}\mu(X_s) + S(X_s) \tag{7}$$

and the present value is

$$v^{\mathrm{LIB}}(t,t+0.25) = E_t^Q \exp\left(-\int_t^{t+0.25} \lambda^{\mathrm{LIB}}(X_s)\,\mathrm{d}s\right). \tag{8}$$

Eq. (6) takes into account the quoting conventions in the LIBOR market so both LIBOR rates and swap rates are calculated using the correct cash market conventions. There are three stochastic components in the determination of LIBOR rates. The first component is the riskless rate $r(X_s)$. The second component, $v_{\mathrm{AA}}\mu(X_s)$, is the loss-adjusted AA intensity of default. If these were the only two components defining LIBOR, we would be working under the assumption that the 3-month LIBOR rate and the yield on a 3-month AA corporate bond are equal. This is an assumption typically used in the literature, see for example Duffie and

Singleton (1997), Collin-Dufresne and Solnik (2001), Liu, Longstaff, and Mandell (2006), and He (2001). However, Duffie and Singleton (1997) note that the assumption—which they call *homogeneous LIBOR-swap market credit quality*—is nontrivial since the default scenarios, recovery rates, and liquidities of the corporate bond and swap markets can differ. The additional component $S(X_s)$, which we use, accounts for such differences and as we will discuss later this component has important consequences for the model's ability to fit swap rates. We assume that the component $S(X_s)$ that allows for differences in swap and corporate bond markets is defined by

$$S(X) = d + X_6.$$

In contrast to the other five factors, $S(X)$ only comes into play in pricing swaps. With the floating-rate payments on the swap in place, we proceed to value the swap, i.e., to find the fixed-rate payments needed to give the contract an initial value of zero. We compute the value of the swap by taking present values separately of the fixed and floating payments, and by discounting both sides of the swap using the riskless rate. This amounts to ignoring *counterparty risk* in the swap contract—a standard assumption in recent papers.[2] From a theoretical perspective this assumption is justified in light of the small impact that counterparty default risk has on swap rates when the default risk of the parties to the swap is comparable as shown in Duffie and Huang (1996) and Huge and Lando (1999). From a practical perspective, posting of collateral and netting agreements reduce—if not eliminate—counterparty risk. Bomfim (2002) shows that even under times of market distress there is no significant role for counterparty risk in the determination of swap rates.

With these assumptions we can value the swap rates in closed form. The swap data in the empirical section are interest rate swaps where fixed is paid semiannually and floating is paid quarterly. We consider an interest rate swap contract with maturity $T-t$, where $T-t$ is an integer number of years. Defining $n = 4(T-t)$ as the number of floating-rate payments at dates $t_1, \ldots, t_n$ and $F(t,T)$ as the $T-t$ year swap rate, the 3-month LIBOR, $L(t_{i-1}, t_i)$, is paid at time $t_i, i = 1, \ldots, n$ while the fixed-rate payments $F(t,T)/2$ are paid semiannually, i.e., at times $t_2, t_4, \ldots, T$. The resulting formula for the swap rate is

$$F(t,T) = \frac{2\sum_{i=1}^{n}(\mathrm{e}^{A^s(t_{i-1}-t)+\mathbf{B}^s(t_{i-1}-t)'X_t} - P(t,t_i))}{\sum_{i=1}^{n/2} P(t,t_{2i})},$$

where the functions A^s and $\mathbf{B}^s$ are found in Appendix B.

3. Data description

The data consist of U.S. Treasury yields, swap rates, and corporate yields for the rating categories AAA, AA, A, and BBB on a weekly basis from December 20, 1996 to December 30, 2005, a total of 472 observations for each time series. The rates are Friday's closing rates.

The Treasury data consist of weekly observations of the most recently auctioned issues adjusted to constant maturities published by the Federal Reserve in the H-15 release. More specifically, in recent years the current inputs are the most recently auctioned 4-, 13- and 26-week bills, plus the most recently auctioned 2-, 3-, 5-, and 10-year notes. The quotes for these securities are obtained at or near the 3:30 PM close each trading day. The Treasury estimates a cubic spline that passes exactly through the yields on those securities, so that the spline is used only to make a small maturity adjustment.[3] We use maturities 1, 2, 3, 5, 7, and 10 years.

Swap rates are taken from Bloomberg and are for swaps with a semiannual fixed rate versus 3-month LIBOR. The rates are means of the bid and ask rates from major swap dealers' quoted rates. Data cover the maturities 2, 3, 5, 7, and 10 years. In addition to the swap data, 3-month LIBOR is used in estimation.

Corporate rates are zero-coupon yields obtained from Bloomberg's Fair Market Yield Curves (FMYC) for banks/financial institutions for the investment grade categories AAA, AA, A, and BBB and cover the maturities 1, 2, 3, 4, 5, 7, and 10 years.[4] The A and BBB curves along with the AA curve in the period

[2]See He (2001), Grinblatt (2001), Collin-Dufresne and Solnik (2001), and Liu, Longstaff, and Mandell (2006).

[3]Further information about the Treasury yield curve methodology can be found on the United States Department of Treasury's web page http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/yieldmethod.html.

[4]For more information and a review of Bloomberg's estimation methodology see Doolin and Vogel (1998) and OTS (2002).

September 21, 2001–December 30, 2005 are based on bonds issued by U.S. banks, while the AAA curve is based on both U.S. banks and financial institutions. The AA curve in the period December 20, 1996–September 14, 2001 is based on financial institutions since Bloomberg does not report a AA corporate curve for banks in this period. The long end of the AAA curve is inconsistent with the rest of the yield curves for extended periods of time after September 14, 2001 and we therefore leave out the 5-, 7-, and 10-year AAA yields after this date and treat the data as missing in the estimation.

The corporate bond yields and the 3-month LIBOR rate are converted to continuously compounded yields before the analysis, and we take into account the money market quoting conventions in the LIBOR market. In total we have 18,208 yield observations: 2,832 Treasury observations, 2,360 swap rate observations, 12,544 corporate bond observations, and 472 LIBOR rate observations.

4. Estimation methods

Similar to Duffee (1999) and Driessen (2005), we estimate the model using both the cross-sectional and time-series properties of the observed yields by use of the extended Kalman filter. Details of the extended Kalman filter are given in Appendix C.

Each week we observe 40 yields (we return to missing observations later):

- Six Treasury par rates
- Seven AAA corporate yields
- Seven AA corporate yields
- Seven A corporate yields
- Seven BBB corporate yields
- One LIBOR rate
- Five swap rates

Recall that $X_t = (X_{t1}, \ldots, X_{t6})'$ where $X_1, \ldots, X_6$ are six independent affine processes. Suppressing the dependence on the parameters, we have the measurement and transition equation in the Kalman filter recursions as

$$y_t = A_t + B_t X_t + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{N}(0, H_t), \tag{9}$$

$$X_t = C_t + D_t X_{t-1} + \eta_t, \quad \eta_t \sim \mathrm{N}(0, Q_t), \tag{10}$$

where $\mathrm{N}(0, \Sigma)$ denotes a normal distribution with mean zero and covariance matrix Σ.

We first set up the transition equation (10). The conditional mean and variance of X_t are linear functions of X_{t-1} (see de Jong, 2000),

$$E(X_t|X_{t-1}) = C + DX_{t-1}, \quad Var(X_t|X_{t-1}) = Q^1 + Q^2 X_{t-1},$$

where the matrices D and Q^2 are diagonal since the processes are independent. We do not observe X_{t-1} and therefore use the Kalman filter estimate $\hat{X}_{t-1}$ in the calculation of the conditional variance, $Q_t = Q^1 + Q^2 \hat{X}_{t-1}$.

The parameters λ_{ij} in Eq. (4) are estimated using Moody's corporate bond default database for the period 1987–2002. The matrix is shown in Table 1 in a generator matrix form where diagonal elements take the form $\lambda_i = -\sum_{j \neq i} \lambda_{ij}$.

We assume that rating transitions are conditionally Markov and ignore downward drift effects. That is, for a given level of $\mu(X_s)$, the intensity of downgrade is a function only of the current state and not of the previous rating history. Results in Lando and Skødeberg (2002) indicate that this is not an unreasonable approximation for financial firms.

As mentioned, the data include corporate yields for the rating categories AAA, AA, A, and BBB, i.e., investment grade ratings. The remaining rating categories BB, B, and C, which are all speculative grade rating

Table 1
The transition intensity matrix (excluding the default state) for corporate bonds estimated using Moody's corporate bond default database for the period 1987–2002. The speculative grade categories are gathered in one state as shown in Table 2 before the matrix is used in the empirical work via the pricing formula (5).

$\tilde{\Lambda}$	AAA	AA	A	BBB	BB	B	C
AAA	−0.0976	0.0847	0.0122	0.0007	0	0	0
AA	0.0157	−0.1286	0.1090	0.0028	0.0003	0.0008	0
A	0.0010	0.0267	−0.1012	0.0678	0.0047	0.0010	0
BBB	0.0009	0.0024	0.0669	−0.1426	0.0647	0.0067	0.0009
BB	0	0.0004	0.0066	0.1220	−0.2391	0.1069	0.0031
B	0	0.0004	0.0024	0.0103	0.0672	−0.2037	0.1233
C	0	0	0.0018	0.0070	0.0070	0.0648	−0.0806

Table 2
The transition intensity matrix intensities from Table 1, where speculative grade states are gathered in one state, SG.

$\tilde{\Lambda}$	AAA	AA	A	BBB	SG
AAA	−0.0976	0.0847	0.0122	0.0007	0
AA	0.0157	−0.1286	0.1090	0.0028	0.0011
A	0.0010	0.0267	−0.1012	0.0678	0.0057
BBB	0.0009	0.0024	0.0669	−0.1426	0.0723
SG	0	0.0004	0.0066	0.1220	−0.1291

categories, are treated as one rating category denoted SG. The generator matrix in Table 1 is therefore reduced in the following way:

- for the investment grade rating categories, the transition intensities for changing rating to BB, B, and C are added and used as the transition for changing rating to SG,
- the intensities for going from BB to investment grade ratings are used as the intensities for going from SG to investment grade. The intensity for a jump to a different rating from SG ($\lambda_{\mathrm{SG,SG}}$) is changed such that the last row in the new generator matrix still sums to zero.

The resulting generator matrix is given in Table 2. The new category SG can be regarded as a ''downward-adjusted'' BB category, because the transition intensities from BB are kept, while the transition intensities to SG are slightly higher than the original transition intensities to BB. The problems of reducing the generator matrix are concentrated in the SG rating category. If we were to price speculative grade bonds, this way of reducing the generator matrix would be problematic, but since we only price bonds rated AAA, AA, A, and BBB the adjusted transition intensities do not cause problems for the modeling.

Observed yields are nonlinear functions of the state variables and we write the relation as $y_t = f(X_t)$. A first-order Taylor approximation of $f(X_t)$ around the forecast $\hat{X}_{t|t-h}$,

$$f(X_t) \simeq f(\hat{X}_{t|t-h}) + \hat{B}_t(X_t - \hat{X}_{t|t-h}) = f(\hat{X}_{t|t-h}) - \hat{B}_t\hat{X}_{t|t-h} + \hat{B}_t X_t,$$

where

$$\hat{B}_t = \frac{\partial f(x)}{\partial x}\bigg|_{x=\hat{X}_{t|t-h}} \tag{11}$$

yields the matrix B_t in the measurement equation. It is not necessary to calculate A_t in the linearization since it is not used in the extended Kalman filter.

We assume that all 40 yields and rates are measured with independent errors with identical variance, so $var(\varepsilon_t) = \sigma^2 I_{40}$. Furthermore, we assume that the processes are stationary under P (implying $k_i<0$) and use the unconditional distribution as the initial distribution in the Kalman filter recursions. This is consistent with empirical evidence in the literature suggesting that Treasury and corporate bond yields might not be stationary under the equivalent martingale measure but exhibit stationarity under the true measure P. As an example, Duffee (1999, p. 199) notes that "for the typical firm, default risk is mean-reverting under the true (physical) measure, but mean-averting (i.e., nonstationary) under the equivalent martingale measure." BBB yields and AAA yields for certain maturities are missing for a period but the Kalman filter can easily handle missing observations and the details are given in Appendix C. The reason for restricting the Q-mean of all the processes $X_1, \ldots, X_6$ to be zero is that not all of the parameters can be estimated empirically. For example, in $r = X_1 + X_2$ the mean and α_i of each factor are not separately identified (see de Jong, 2000). With this normalization α can be interpreted as the average volatility of each factor. In addition, we add a constant mean to the processes describing the government rate, the convenience yield, the default and rating adjustment process μ, and the swap factor S processes. In summary, we have the following model:

$$r^{g}(X) = a + X_1 + X_2, \tag{12}$$

$$r(X) = a + X_1 + X_2 + (e + X_5), \tag{13}$$

$$\mu(X) = b + X_3 + X_4 + c(X_1 + X_2), \tag{14}$$

$$S(X) = d + X_6. \tag{15}$$

Restricting D to be a positive CIR process implies the restriction $\alpha_5 = e\beta_5$.

The outlined extended Kalman filter does not yield consistent parameter estimates for two reasons. First, in the estimate of $Var(X_t|X_{t-1})$ we use $\hat{X}_{t-1}$ instead of X_{t-1} and set $\hat{X}_{it} = -\alpha_i/\beta_i$ if $\hat{X}_{it} < -\alpha_i/\beta_i$. Nevertheless, Monte Carlo studies in Lund (1997), Duan and Simonato (1999), and de Jong (2000) indicate that the bias is small. Second, the pricing function f in the measurement equation is linearized around $\hat{X}_{t|t-1}$. A Monte Carlo study in an earlier version of this paper shows that the bias due to the linearization is also small and the results are available on request.

5. Empirical results

Before we turn to the main results of the paper we examine our model along several dimensions to check whether the implications of the model are consistent with key features of the data. In this section we look at the average pricing errors of the model and compare our estimated parameters with findings in previous studies. We also interpret the latent variables, and to justify our interpretation of the Treasury convenience yield we compare the estimated riskless rate at the 10-year maturity with the yield on 10-year Fannie Mae bonds. Finally, we compare the credit risk component in corporate bond yields with spreads in the credit default swap market.

To assess the model's simultaneous fit to all the curves, Table 3 shows the mean, standard error, and first-order autocorrelation of the residuals. The maximum average pricing error for any yield is less than eight basis points, and for the Treasury and swap yields the maximum average pricing error is 3.25 basis points. The BBB yield curve has the worst fit, which is seen by the largest average standard errors. This suggests that our specification of the generator matrix enables us to accurately price highly rated corporate bonds, while the pricing of lower-rated bonds might be more problematic. However, for our purpose the fit of the corporate bonds is satisfactory. The yield curve with the smallest average standard errors is the swap curve where the average error is 7.4 basis points, which is comparable to average standard errors in other papers estimating swap rates such as 6.1 bp in Duffie and Singleton (1997), 10.7 bp in Dai and Singleton (2000, Table IV, $A_2(3)_{DS}$), 4.5 bp in Collin-Dufresne and Solnik (2001), and 7.1 bp in Liu, Longstaff, and Mandell (2006). However, comparison of standard errors should be done with caution since the number of factors and the data are different from paper to paper. We note that a sign of misspecification of the model is that the first-order correlations of the residuals are strongly positive, which is also found in other papers such as Duffie and Singleton (1997), Dai and Singleton (2000), and Collin-Dufresne and Solnik (2001).

Table 3
Statistics for the pricing errors of the Treasury, corporate, LIBOR, and swap rates measured in basis points. The pricing error is $\varepsilon = y_t - \hat{y}_t$ where $\hat{y}_t$ is the model-implied yield and y_t the observed yield. The means, standard deviations, and first-order autocorrelations ρ are shown.

	$\varepsilon_{0.25}$	ε_1	ε_2	ε_3	ε_4	ε_5	ε_7	ε_{10}	Average
Govt									
Mean		−0.51	2.69	−1.73		−2.94	3.01	−1.48	−0.16
St. dev.		9.35	9.03	8.31		4.1	7.24	11.78	8.3
ρ		0.913	0.944	0.936		0.782	0.898	0.951	0.904
AAA									
Mean		−5.29	−1.43	3.11	3.93	−1.64	0.16	−4.44	−0.8
St. dev.		7.96	8.27	6.3	7.23	5.45	7.48	10.53	7.6
ρ		0.834	0.893	0.827	0.879	0.73	0.755	0.868	0.827
AA									
Mean		1.02	0.99	1.95	1.88	−4.11	7.48	−3.98	0.747
St. dev.		6.93	7.67	9.74	10.71	8.67	13.8	10.36	9.7
ρ		0.736	0.84	0.88	0.911	0.821	0.934	0.875	0.857
A									
Mean		−2.49	−2.69	0.35	1.86	−6.15	6.82	0.18	−0.3029
St. dev.		7.93	9.47	7.61	8.11	8.2	9.74	13.3	9.19
ρ		0.808	0.901	0.861	0.894	0.88	0.91	0.928	0.883
BBB									
Mean		2.6	−0.41	−2.04	0.84	−5.82	7.78	−2.55	0.0571
St. dev.		10.19	8.99	9.9	10.36	10.74	13.42	14.89	11.21
ρ		0.851	0.885	0.865	0.923	0.912	0.922	0.897	0.894
LIBOR									
Mean	5.81								5.81
St. dev.	22.03								22.03
ρ	0.914								0.914
Swap									
Mean			−3.25	−1.49		−1.52	−1.03	1.75	−1.108
St. dev.			12.04	9.3		4.98	4.15	6.71	7.44
ρ			0.963	0.939		0.76	0.717	0.82	0.84

Next, we report the estimated parameters in Table 4. Two sets of standard errors are reported: White (1982) heteroskedasticity-corrected standard errors and standard errors without the correction. The former is theoretically more robust while the latter is numerically more stable and details are given in Appendix D.

The means of the variables are difficult to estimate reliably and therefore are hard to interpret, which is a common problem (see, e.g., Duffee, 1999; Duffee and Stanton, 2001). From the table we see that credit risk has a weak positive dependence on government rates because the parameter c is positive but statistically insignificant. The literature is divided on this dependence. Research using only Treasury and corporate bond data (Duffee, 1999; Driessen, 2005) shows a negative dependence. Collin-Dufresne and Solnik (2001) include swaps in the estimation and also find a negative dependence, whereas Liu, Longstaff, and Mandell (2006) use only swap and Treasury data and show a positive dependence.

Turning to the filtered state variables, linear combinations of the two variables X_1 and X_2 have the usual interpretation as the level and slope of the Treasury curve as seen in Fig. 2. Given the specification of the short rate as $r^g(X) = a + X_1 + X_2$, it is not surprising as seen in the figure that a (translated) sum of X_1 and X_2 can be interpreted as the level of the government yield curve, but we also see that a rotation of X_1 and X_2 accurately tracks the slope of the Treasury yield curve. This shows that the estimation results for the Treasury curve in our joint estimation are similar to what we would expect had we only estimated a two-factor model for the Treasury curve.

Table 4
Parameter estimates resulting from the Kalman filter estimation. The first set of standard errors is calculated as

$$\hat{\Sigma}_1 = \frac{1}{T}[\hat{A}\hat{B}^{-1}\hat{A}]^{-1},$$

where $\hat{A} = -(1/T)\sum_{i=1}^{T}(\partial^2 \log l_t(\hat{\theta})/\partial\theta\partial\theta')$ and $\hat{B} = (1/T)\sum_{i=1}^{T}(\partial \log l_t(\hat{\theta})/\partial\theta)(\partial \log l_t(\hat{\theta})'/\partial\theta)$. The second set of standard errors is calculated as

$$\hat{\Sigma}_2 = [T\hat{B}]^{-1}.$$

Note that there are no standard errors on α_5 because of the restriction $\alpha_5 = e\beta_5$ ensuring that $D(X) = e + X_5$ remains positive.

Parameters of the state variables

	k	θ	α	β	λ	$k*$
X_1	−0.1368	−0.04865	0.0009339	0.01170	−7.129	−0.05344
			(0.000064)	(0.000904)	(2.562)	(0.002997)
			(0.000200)	(0.001684)	(40.23)	(0.010715)
X_2	−0.3756	−0.01194	0.0002139	$3.29 * 10^{-6}$	−20.97	−0.37550
			(0.000037)	(0.001090)	(12.30)	(0.012165)
			(0.000036)	(0.001005)	(56.55)	(0.005407)
X_3	−0.2805	−0.4380	0.3198	0.00089	−0.3842	−0.28015
			(0.0434)	(0.00007)	(0.3643)	(0.00092)
			(0.0827)	(0.00058)	(1.098)	(0.00009)
X_4	−0.4281	−0.7725	0.4900	0.61085	−0.6750	−0.01583
			(0.1699)	(0.03975)	(0.0058)	(0.00001)
			(0.7107)	(0.05726)	(0.2319)	(0.00435)
X_5	−0.1103	−0.0173	$1.00 * 10^{-5}$	0.00050	−190.8	−0.01458
				(0.00004)	(80.76)	(0.00284)
				(0.00024)	(545.6)	(0.01657)
X_6	−0.0036	−0.0088	$8.07 * 10^{-6}$	0.00091	−3.948	$-2.99 * 10^{-5}$
			$(0.28 * 10^{-6})$	(0.00004)	(0.278)	$(1.88 * 10^{-5})$
			$(2.2 * 10^{-6})$	(0.00048)	(136.7)	$(3.75 * 10^{-11})$

Other parameters

a	b	c	d	e	σ^2
0.09324	0.98220	1.3019	0.00487	0.01998	$1.18 * 10^{-6}$
(0.00374)	(0.23043)	(1.0149)	(0.00233)	(0.00024)	$(0.04 * 10^{-6})$
(0.00926)	(1.0563)	(3.6600)	(0.01616)	(0.00425)	$(0.01 * 10^{-6})$
v_1	v_2	v_3	v_4	v_5	
2.20e − 06	1.08e − 03	4.65e − 03	8.01e − 03	1.71e − 02	
$(22.6 * 10^{-5})$	(0.20e − 03)	(0.36e − 03)	(0.73e − 03)	(0.10e − 02)	
$(7.51 * 10^{-12})$	(0.15e − 03)	(0.54e − 03)	(0.97e − 03)	(0.22e − 02)	

Model
$r^{\text{govt}} = a + X_1 + X_2$
$r^{\text{riskless}} = r^{\text{govt}} + e + X_5$
$\mu = b + c(X_1 + X_2) + X_3 + X_4$
$\lambda^{\text{LIBOR}} = \lambda^{\text{AA}} + d + X_6$

The variables X_3 and X_4 relate to the credit risk component in corporate bond yields. To assess whether the dynamics of the two variables are consistent with actual credit risk dynamics in corporate yields, we compare model-implied credit risk components with actual credit risk components. Since the actual credit risk component is not directly observable, we define it as the actual corporate bond yield minus the estimated riskless rate. Fig. 3 shows the average credit slope and short credit spread. From the top graph we see that the credit slope is captured well in the model, and the bottom graph likewise shows that the short credit spread is

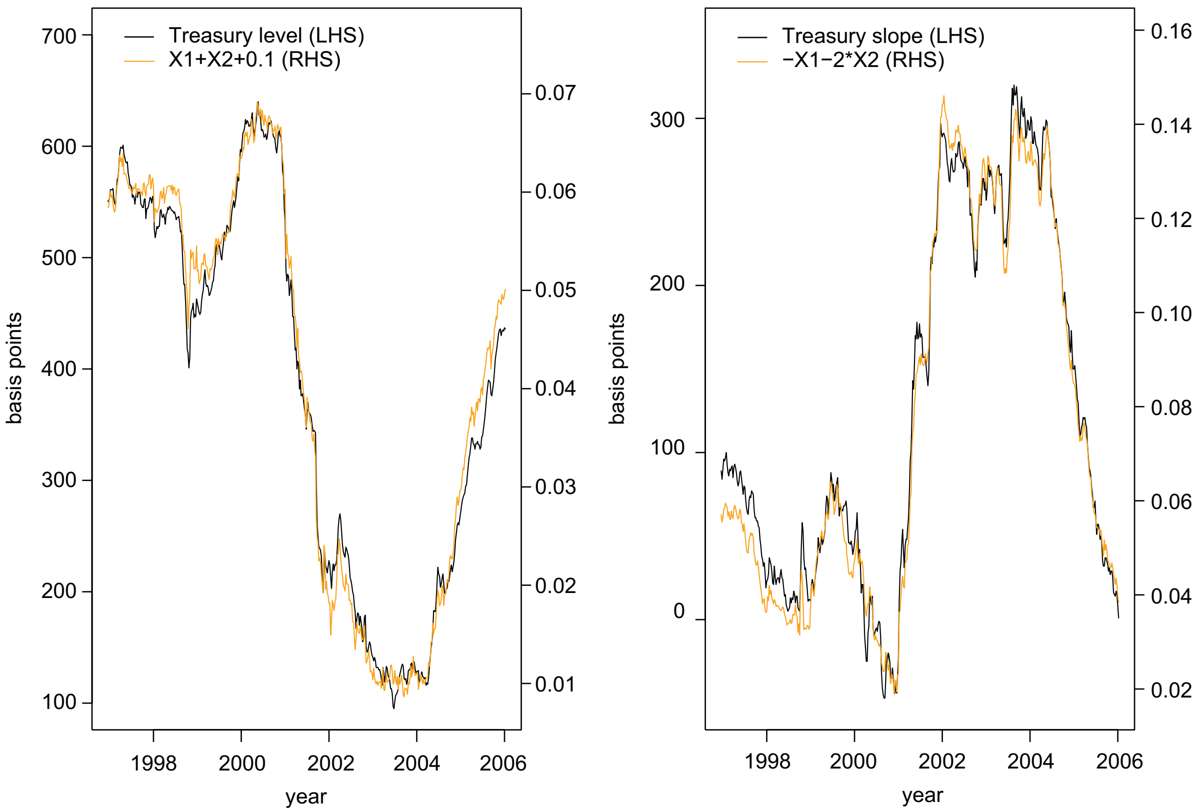

Fig. 2. Interpreting linear combinations of the latent factors driving Treasury yields as the level and slope of the yield curve. The Treasury short rate is given as $r^g(X) = a + X_1 + X_2$ in the model. The observed level is defined as the 1-year constant maturity Treasury (CMT) rate while the slope is the 10-year minus 1-year CMT rate.

measured reasonably well. The specification of μ is therefore flexible enough to capture both the levels and slopes of the credit risk component in corporate bond yields. While not shown in Fig. 3, the dynamics of the four individual credit curves are also captured well by the model.

Finally, we compare our estimates with other sources of data not used in the estimation. Our primary concern is whether our predictions for the convenience yield in long-maturity bonds are reasonable. In the short end of the yield curve, the model predicts that most of the spread between highly rated corporate bonds and government bonds is due to the Treasury convenience yield, which seems reasonable given the extremely small number of historical defaults of highly rated issuers over short periods. However, over longer periods of time highly rated issuers can be downgraded and then default and therefore the relative size of the convenience yield in corporate spreads for longer maturities is less clear. For longer-term maturities we can estimate the effect in basis points of the Treasury factor since the price of a riskless bond is given as

$$P(t,T) = E_t\left(\exp\left(-\int_t^T r^g(s) + L(s)\,ds\right)\right) = P^g(t,T)E_t\left(\exp\left(-\int_t^T L(s)\,ds\right)\right),$$

and therefore the riskless rate is

$$y(t,T) = y^g(t,T) - \frac{1}{T-t}\log\left(E_t\left(\exp\left(-\int_t^T L(s)\,ds\right)\right)\right). \tag{16}$$

Longstaff (2004) suggests using the spread between Refcorp bonds/strips and the government curve as a proxy for the Treasury premium. He notes that there are measurement errors in the data making this spread a noisy estimate of the convenience yield in Treasuries, and an inspection of the spread suggests that the measurement error has become so large in recent years that the spread is too noisy to serve as a proxy for the Treasury

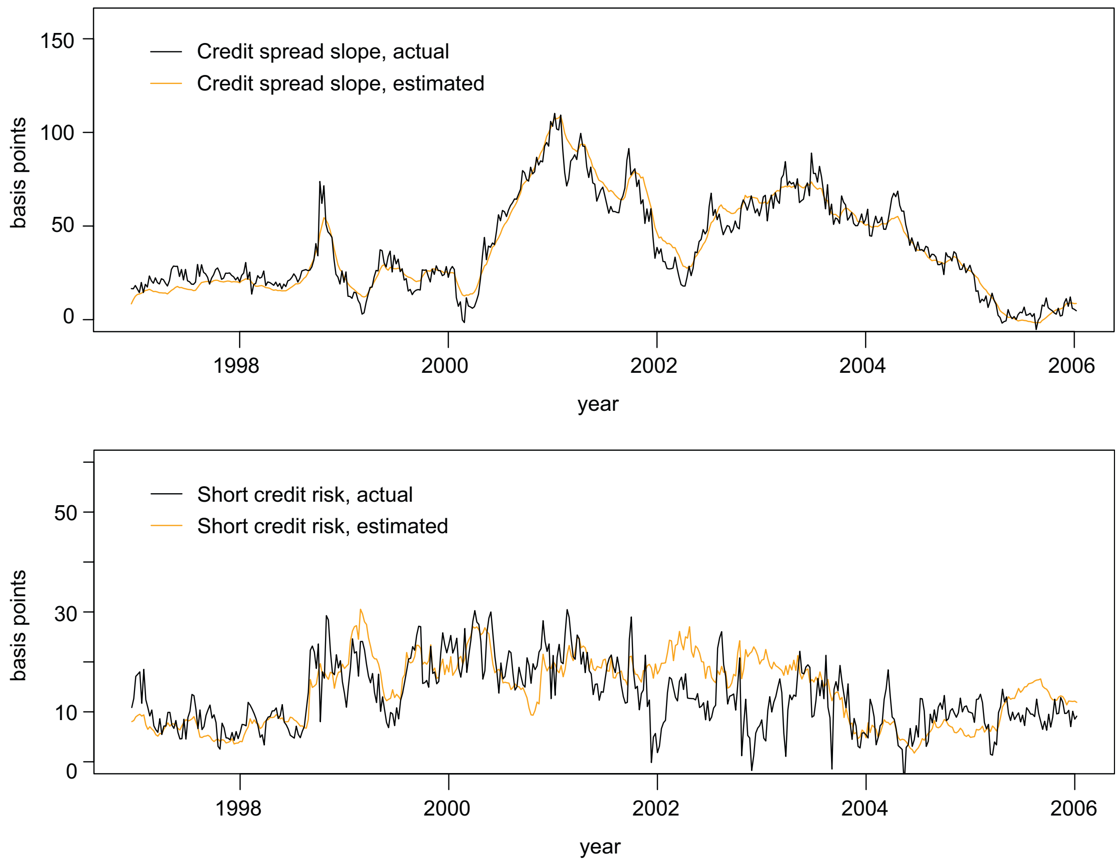

Fig. 3. Interpreting linear combinations of the latent factors driving credit spreads. The first graph shows the average actual and estimated credit spread slopes. The credit spread slope is calculated as the average 10-year AAA, AA, A, and BBB spread to the riskless rate minus the average 1-year AAA, AA, A, and BBB spread to the riskless rate. In derivation of the actual credit spread slope actual corporate yields and estimated riskless rates are used. The second graph shows the actual and estimated short credit spreads. The short credit spread is calculated as the average 1-year AAA, AA, A, and BBB spread to the riskless rate. Again, in the derivation of the actual short credit spread actual corporate yields and estimated riskless rates are used.

convenience yield. For example, the 1-year Refcorp-Treasury spread is negative from May 14, 2004 to May 27, 2005 with an average negative spread of 24.2 basis points. Also, in 2004 and 2005 there are 8 weeks where all Refcorp-Treasury spreads up to 7 years are negative.

As an alternative measure of the 10-year convenience yield, we suggest using the spread between Fannie Mae bonds and Treasury bonds. Fannie Mae is a government sponsored enterprise and can be viewed as having an implicit government guarantee of its debt obligations (see, e.g., Jaffee, 2003). Consequently, the credit risk on Fannie Mae bond issues is small. In addition, Fannie Mae issues debt in large amounts (2,952 billion in 2005) and on a regular basis and daily yield curves are available on its webpage. Fig. 4 shows the 10-year Fannie Mae yield and estimated riskless rate along with the 10-year Treasury and AA corporate rate. The estimated riskless rate tracks the Fannie Mae yield quite closely—except for two periods in 2004 and 2005—and both yields are above the Treasury yield and below the corporate yield. The average Treasury, Fannie Mae, estimated riskless, and corporate AA rates are 5.05%, 5.64%, 5.65%, and 6.11%.

While a formal examination of possible benchmarks for the risk-free rate is outside the scope of this paper, the evidence suggests that the convenience yield is reasonably estimated.[5]

[5]We do not necessarily argue that the Fannie Mae yield curve is a good proxy for the risk-free yield curve. Ambrose and King (2002) find an insignificant repo specialness effect in the 10-year Fannie Mae yield but a significant effect in shorter maturities suggesting that the

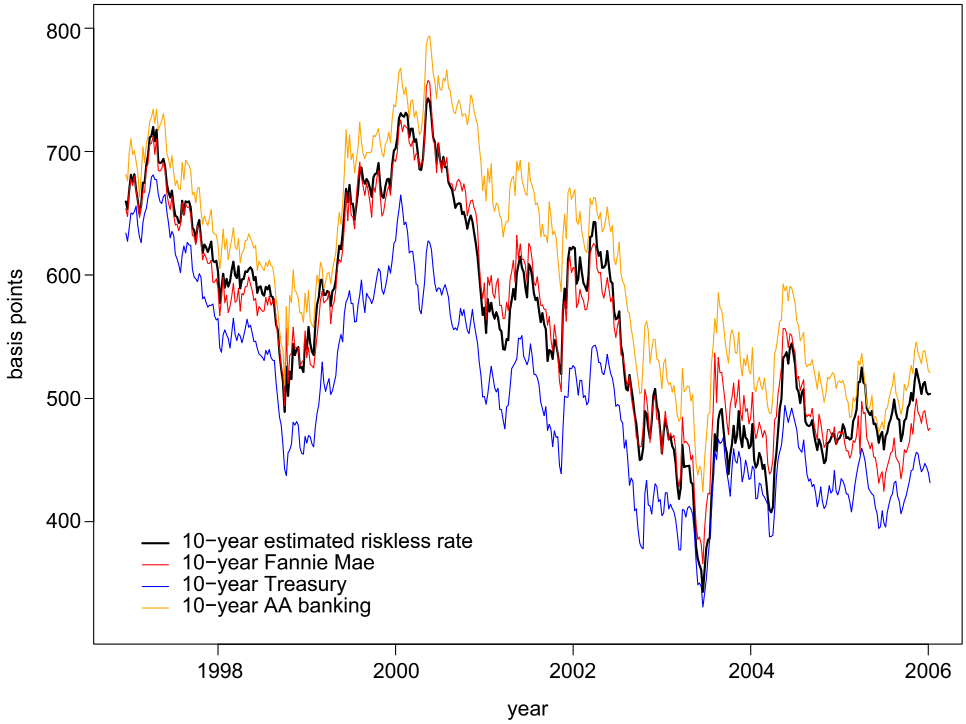

Fig. 4. Comparing the estimated 10-year riskless rate with the observed 10-year yields for Treasury, Fannie Mae, and AA banking bonds. All yields are continuously compounded zero-coupon yields and the data for Fannie Mae bonds are from the Fannie Mae website.

Finally, we compare the model-implied credit risk component with evidence from the credit default swap (CDS) market. If CDS spreads are pure measures of credit risk, as argued by Longstaff, Mithal, and Neis (2005), they should match model-implied credit risk components. Wells Fargo & Co had a stable AA rating throughout the estimation period and their bonds have been an important component of the AA corporate yield curve and therefore their CDS spreads should be comparable to the AA credit risk component.[6] We obtain their 5-year CDS spread, the most liquid and reliable maturity, from MarkIt for the period October 26, 2001–December 30, 2005. The average CDS spread is 23.6 basis points while the average credit risk component in AA spreads is 24.7 basis points, and the correlation between the two series is 85%. This evidence further indicates that the credit risk component is well measured.

In summary, our model has small average pricing errors. We have interpreted the first two latent factors as the level and slope of the Treasury curve, and shown that both the short spread and slope of the credit risk component in corporate yields are captured by the model. We have also found the estimated 10-year riskless rate to be consistent with the 10-year Fannie Mae yield, and therefore our latent Treasury convenience yield factor seems well specified. Evidence from the CDS market suggests that the credit risk components in corporate yields are well estimated. We will return to the interpretation of the sixth factor—the swap factor—below.

5.1. AA credit risk and the LIBOR-GC repo spread

As noted in Collin-Dufresne and Solnik (2001), the refreshed nature of the LIBOR rate used for fixing the floating-rate payment of an interest rate swap implies that the contribution of credit risk to the swap curve

(*footnote continued*)
short end of the Fannie Mae yield curve has stronger repo specialness effects than the long end. In March 1998 Fannie Mae started a program of creating benchmark securities which might have amplified the effects found in their study.

[6]On March 20, 2006 there were 35 bonds underlying the AA curve and of these six were issued by Wells Fargo, including the bonds with the shortest and longest time to maturity.

comes from the uncertainty in the future credit risk of short LIBOR rates. We now estimate this credit risk contribution across the term structure of swap spreads. Before doing so, we compare our estimated level of AA credit risk with a commonly used proxy for AA credit risk: the spread between LIBOR and GC repo rates. We refer to this spread as the LGC spread.

LIBOR rates are rates on unsecured loans between counterparties rated AA on average and GC repo rates are rates on secured loans; the difference is thought to be due to a credit risk premium. In Fig. 5 we compare the 3-month LGC spread with the estimated 3-month AA credit risk premium. The 3-month AA credit risk premium on date *t* is calculated as the difference in basis points between the yields on a 3-month AA corporate bond and a 3-month riskless bond (with no convenience yield), while the 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated premium is 4.6 basis points while the average observed LGC spread is 12.4 basis points. Furthermore, the LGC spread is very volatile while the estimated AA default premium is much more persistent. A possible explanation for the different behavior of the two time series is given in Duffie and Singleton (1997). In their model the LIBOR rate is poorly fitted and they suggest that there might be noncredit factors determining LIBOR rates. Support for this view is given in Griffiths and Winters (2005), who examine 1-month LIBOR and find a turn-of-the-year effect: the rate increases dramatically at the beginning of December, remains high during December, and decreases back to normal at the turn of the year, with the decline in rates beginning a few days before year-end. This effect is most likely a liquidity effect unrelated to credit risk, and if the GC repo rate does not have a turn-of-the-year effect, the LGC spread will mirror this liquidity effect.

Furthermore, we see the largest difference between the LGC spread and the estimated AA credit premium in the last 3 months before the millenium date change (Y2K). Three months before Y2K, the LGC spread (based on 3-month LIBOR) jumps from 11 to 79 basis points. The spread between 2-month LIBOR and GC repo jumps two months before Y2K, and the 1-month spread jumps one month before Y2K, as seen in Fig. 6. As argued in Sundaresan and Wang (2006), lenders in the interbank market wanted a premium to lend cash due

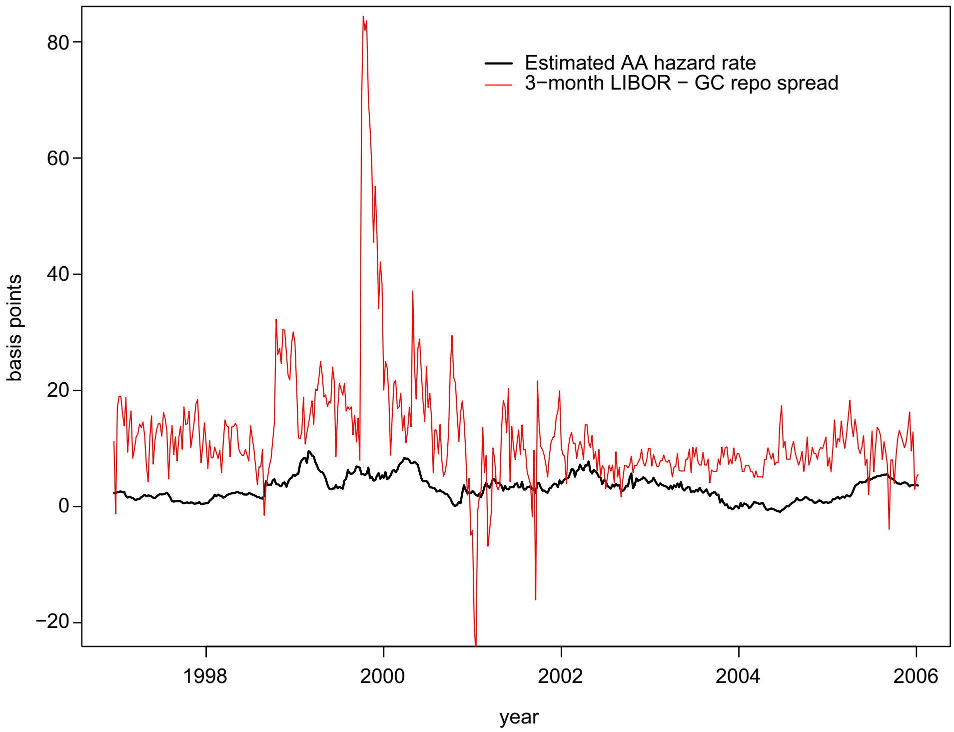

Fig. 5. Comparing the estimated 3-month AA credit risk spread with the observed 3-month LIBOR-GC repo spread (denoted the LGC spread). The 3-month AA credit risk spread is calculated as the difference between the yields on a 3-month AA corporate bond and a 3-month riskless bond. The 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated 3-month AA credit spread is 3.2 basis points, while the average LGC spread is 12.4 basis points.

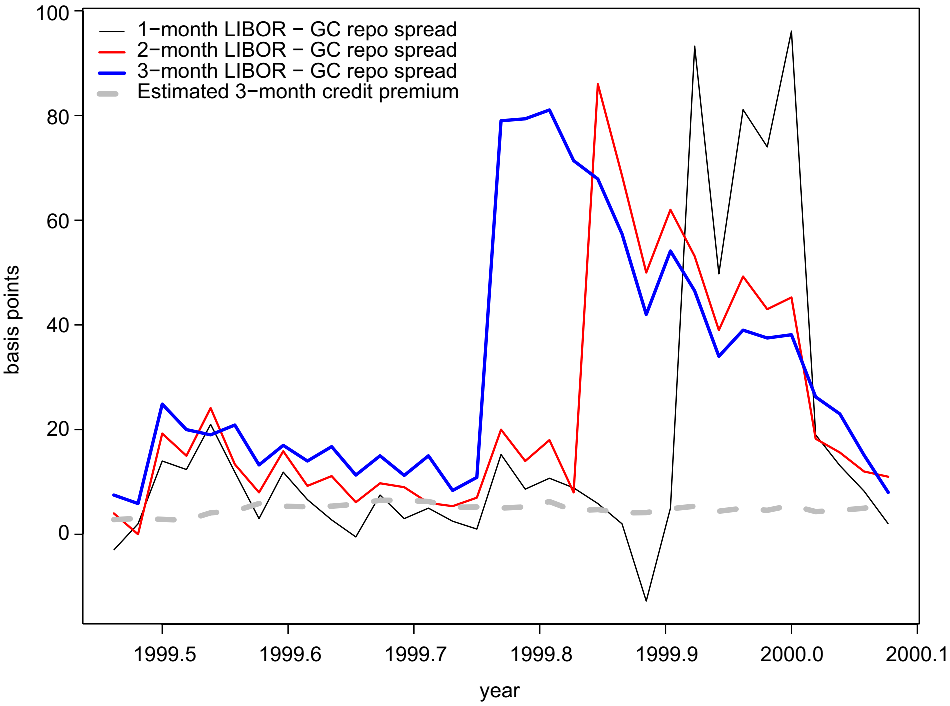

Fig. 6. The 1-, 2-, and 3-month LIBOR-GC repo spreads along with the estimated 3-month AA credit spread before Y2K. The 3-month AA credit spread is calculated as the difference between the yields on a 3-month AA corporate bond and a 3-month riskless bond. LIBOR and GC repo rates are from Bloomberg.

shortly after Y2K which is consistent with the jump being due to a liquidity premium on short-term lending. We see no effect in the same period on AA corporate rates, which would have to be the case if the market had concerns with credit risk around Y2K. Also, we see the spread between 3-month LIBOR and GC repo rates tightening as we approach Y2K, and this is not consistent with the spikes in the shorter LIBOR rates being due to credit risk concerns. Because the 3-month credit risk premium in our model is estimated using a range of yields and maturities, we see in Fig. 6 that the premium is practically unaffected by Y2K.

Liu, Longstaff, and Mandell (2006) and Li (2004) proxy credit risk with the LGC spread. Therefore, the credit risk component inherits the properties of the LGC spread in being volatile and rapidly mean-reverting. This in turn implies that long-term swap spreads are only weakly affected by fluctuations in the credit risk component. The credit spread fluctuations in our model are not as mean-reverting and therefore cause larger fluctuations in long-term swap spreads.

The evidence in this section suggests that the LGC spread is an inappropriate proxy for credit risk. Although the floating-rate leg in the swap contract is directly tied to 3-month LIBOR, short-term liquidity effects in the LGC spread imply that this spread is not suitable for catching the credit risk premium in longer-term swaps. Below, we consider the contribution to the swap spread of the credit spread estimated using information from corporate bond spreads.

5.2. Swap rates and mortgage-backed security hedging

The introduction of a convenience yield for holding Treasuries allows us to fit the government bond yield curve and the corporate bond curves simultaneously with reasonable accuracy. However, we cannot fit the swap curve working with the riskless rate and the AA corporate bond spread alone, and the introduction of an idiosyncratic swap factor in our model allows us to assess the importance over time of other factors influencing swap spreads besides the credit risk inherent in LIBOR rates and the convenience yield in Treasuries. To assess the impact of the swap factor, Fig. 7 shows the absolute value of the swap factor throughout our sample period.

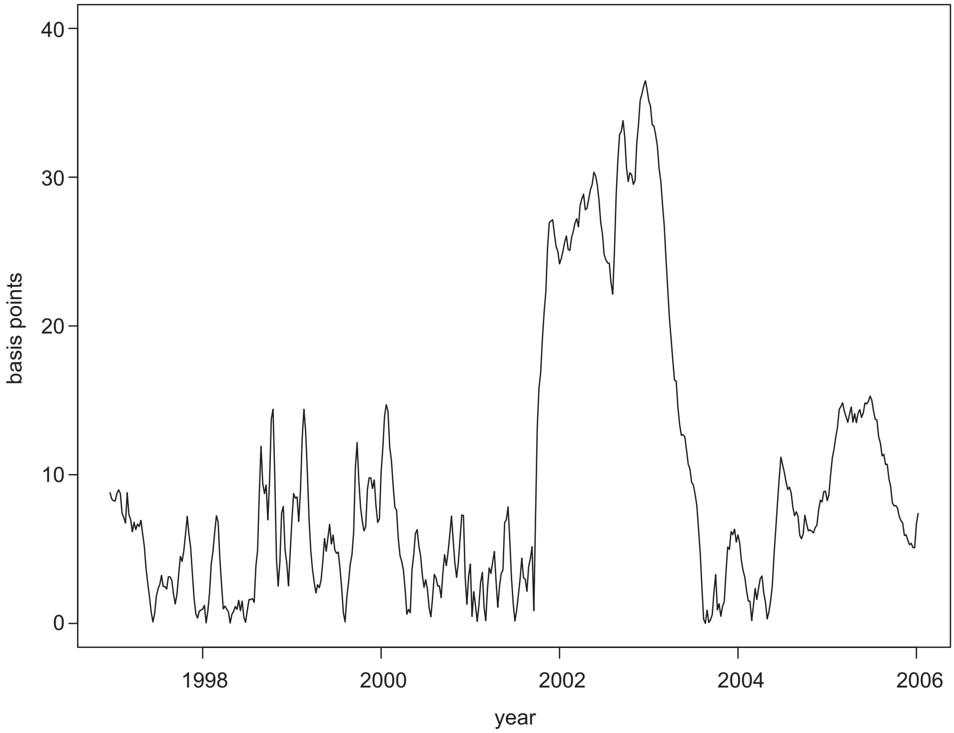

Fig. 7. The absolute value of the estimated swap-specific factor in the swap market. We relax the assumption of *homogeneous LIBOR-swap market credit quality*, i.e., that short-term AA credit spreads are the same whether measured in the corporate bond market or the interbank market. The swap factor captures differences in the two measures of AA credit spreads due to differences in, for example, default scenarios, recovery rates, and liquidity.

For the whole period, the average absolute deviation from zero is 9.3 basis points, but there are large deviations from September 2001 to August 2003. While the maximal deviation from zero in the sample period without those two years is 14.9 basis points, we see that the factor deviates strongly from zero during those 2 years, reaching an absolute deviation of 36.1 basis points on December 13, 2002. Without the 2 years the average absolute deviation is 5.5 basis points. This result suggests that the assumption of homogeneous LIBOR-swap market credit quality is not an unreasonable assumption in calm periods of the swap market, but the relation is violated during stressful periods.

While there might be several reasons for the difference in the corporate and swap markets, we argue that an important factor is MBS hedging. Before turning to the evidence, we briefly recall why there is likely to be a connection between the two markets. Due to the prepayment risk embedded in MBSs—borrowers are allowed to prepay the mortgage which creates uncertainty regarding the timing of cash flows of MBSs—movements in interest rates often result in significant changes in the option-adjusted duration of an MBS. When interest rates drop, borrowers can refinance their mortgages by exercising their option to call the mortgages at par value. This causes a fall in the duration of mortgage-backed securities. Hedging activity in connection with this duration change has the potential for creating large flows in the fixed income markets. The U.S. mortgage market has more than doubled in size since 1995 and in 2000 it surpassed the Treasury market (see BIS, 2003b). Furthermore, as Wooldridge (2001) notes, non-government securities were routinely hedged with government securities until the financial market crisis in 1998. However, periodic breakdowns in the normally stable relation between government and non-government securities led many market participants to switch hedging instruments from government to non-government securities such as interest rate swaps. Today, interest rate swaps and swaptions are the primary vehicles for duration hedging of MBS portfolios, a point also confirmed in studies by Perli and Sack (2003), Duarte (2005), and Chang, McManus, and Ramagopal (2005), all of which are primarily concerned with the volatility effects of this hedging. BIS (2003b) argues that the concentration of OTC hedging activity in a small number of dealers in the swap market seems to make the market more vulnerable to a loss of liquidity. If a few dealers breach their risk limits and cut back on their market-making activity, the whole market loses liquidity. Falling option-adjusted durations can thus cause swap rates to fall below their long-run level and vice versa.

To illustrate that there is a relation between swap and MBS markets, Fig. 8 shows periods of strong refinancing activity in the MBS market along with the swap factor. Mortgage originations can be due to both the purchase of homes and from refinancing existing loans, and the dollar amount of mortgage originations due to refinancing is frequently mentioned to in market commentary. Perli and Sack (2003) provide a more thorough discussion of this data series.

For many of the periods we see the expected relation between refinancing and the swap factor: in quarters with considerable refinancing the swap factor is decreasing. For example, in the last quarter in 1998, which is the first quarter in the sample with high refinancing activity, the swap factor falls 16.6 basis points.

In the second half of 2001, a long period of high refinancing activity begins. As noted in BIS (2002a, p. 40), "The sharp decline in long-term interest rates between June and early November (2001) led to a surge of mortgage refinancing and consequently to a shortening of the duration of MBS portfolios. This decline prompted market participants to seek fixed-rate payments through swaps and swaptions." In our model, the effect on the 10-year swap rate from the end of June to the beginning of November arises because the swap factor fell 22.2 basis points, credit risk fell 1.0 basis point, and the convenience yield rose 9.0 basis points consistent with the view that hedging was the dominant factor in the narrowing of swap spreads in this period.

In the first half of 2002, modest refinancing activity predicts that the swap factor should be increasing, but in this period there might have been other factors more important to the swap factor than hedging. As BIS (2002b) reports, a steep corporate yield curve combined with the fact that many corporate issuers lost access to the commercial paper market led many corporate borrowers to issue long-term fixed debt and swapping into short-term floating debt by entering swap contracts as fixed-rate receivers. BIS (2002b, p. 7) also notes that "spreads of five-year swap yields over U.S. Treasury yields narrowed by 22 basis points during the first four

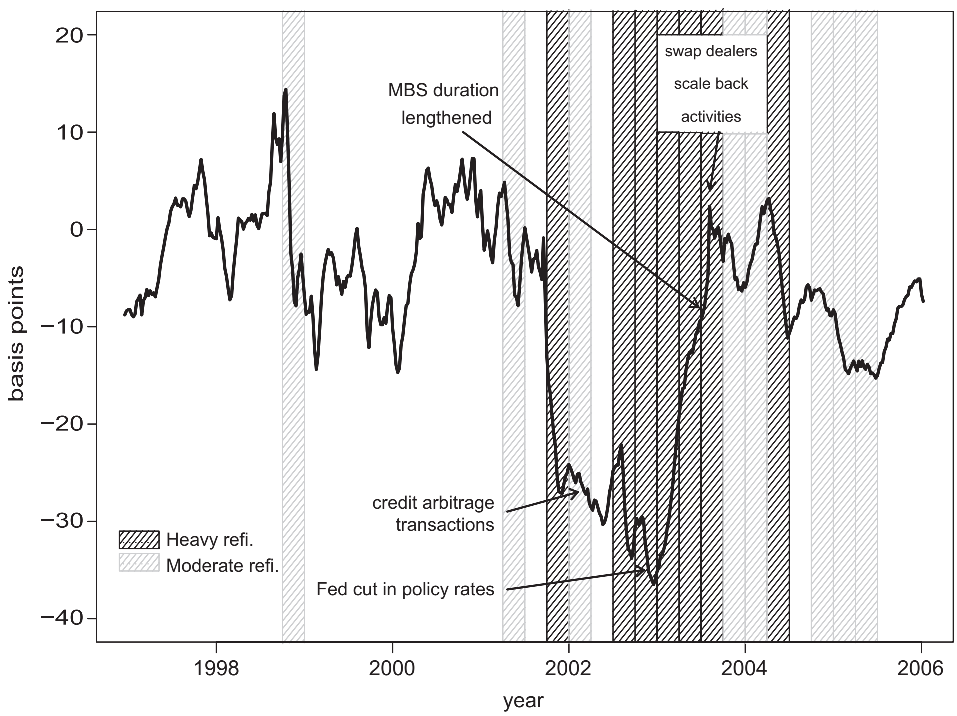

Fig. 8. The estimated swap-specific factor, periods of strong refinancing in the MBS market, and other events that have affected the swap market. Periods of heavy refinancing are quarters with refinancing of more than $400 billion while periods of moderate refinancing are quarters with refinancing of $300–$400 billion. Source: Mortgage Bankers Association at www.mbaa.org.

months of 2002, in part because of such credit arbitrage transactions.'' From the end of 2001 until the end of April 2002, the swap factor fell a modest 4.3 basis points while credit risk fell 2.3 basis points and the convenience yield fell 11.7 basis points. These results suggest that the majority of the fall in swap spreads was due to a lower convenience yield in Treasuries while credit arbitrage transactions and MBS hedging might have influenced the swap factor in opposite directions.

In December 2002 the swap factor reaches the lowest level during the estimation period and although refinancing activity remains high in the following months the imbalance in the swap market is slowly diminishing. Contributing to a return to normal levels of the swap factor despite continued refinancing is a surprise rate cut by the Federal Reserve on November 6, 2002. BIS (2003a, p. 39) notes that ''the spike in U.S. mortgage refinancing observed in early October continued to support the use of swaps to hedge prepayment risk. However, this increase in hedging transactions was probably followed by a moderation in position-taking in the wake of Federal Reserve's surprisingly large cut in policy rates in early November. Following the cut, market participants expected fixed income markets to remain stable for the forthcoming months.''

While the rate cut by the Federal Reserve contributes to an increasing swap factor in the first half of 2003, a surge in long-term yields in June and July 2003 abruptly lengthens the duration of MBSs and as BIS (2003b, p. 7) reports, ''swap-markets tended to become one-sided: sell orders elicited lower prices, and lower prices in turn elicited more sell orders.'' Even though heavy quarterly refinancing in the middle of 2003 predicts a low swap factor, the largest positive change of 7.0 basis points in the swap factor over the estimation period happens in the last week of July/first week of August, making the swap factor positive for the first time in more than 2 years. BIS (2003b, p. 7) writes that ''Mortgage-related markets were especially volatile in the last few days of July and the first few days of August. The widening of swap spreads had caused a number of swap dealers to breach their market risk limits, and they subsequently scaled back their activities. Given the dominance of the swap market by a few dealers, this quickly caused liquidity conditions to deteriorate.''

The detection of idiosyncratic movements in the swap curve is made possible by the inclusion of corporate bond yields in our model, and the first week of August 2003 provides a nice illustration of the effect of adding corporate bonds in the analysis of swap spreads. As previously mentioned the swap factor changes 7.0 basis points in that week while the other two factors influencing swap spreads—credit risk and the convenience yield—change by less than a basis point. This indicates that swap spreads change this week while the spreads between Treasury and corporate yields remain stable, and BIS (2003b, p. 8) confirms that ''The relative lack of movement in the credit markets testifies to the technical nature of the widening of swap spreads in late July. Corporate bond investors appear to have recognized that the phenomenon was driven largely by mortgage hedging and did not reflect an increase in overall credit risk.''

Overall, the evidence in this section shows that the homogeneous LIBOR-swap market credit quality assumption is reasonable for most of the sample but in the period September 2001–August 2003 there are important differences in the two markets. Fig. 8 shows the relation between swap rates and refinancing activity and points to certain periods where other events in the financial markets offset this relation.

5.3. *Decomposing swap spreads*

We have separately analyzed three components in swap spreads—a Treasury convenience yield, a credit risk, and a swap component—and now turn to the joint effect of these components on swap spreads.

In Fig. 1 the estimated ten-year swap spread is decomposed into the three components. The effect of the swap-specific factor at time t is calculated as $-\frac{1}{10}\log(E_t(\exp(-\int_t^{t+10} S(u)\,du)))$. The size of Treasury convenience yield at time t is calculated as $-\frac{1}{10}\log(E_t(\exp(-\int_t^{t+10} e + X_5(u)\,du)))$. The size of the LIBOR credit risk factor at time t is calculated as the difference between the estimated 10-year swap spread and the sum of the Treasury and swap factor at time t. The effects are transformed to basis points in par rates. Throughout the period the Treasury factor accounted for the largest part of the swap spread and it peaked in the middle of 2000 reaching a maximum of 88.9 basis points. The swap factor accounted for a relatively small part of the swap spread before 2000, while it contributed to a contraction of the swap spread from late 2001 to 2003.

The credit risk factor has a relatively small impact on the 10-year swap spread prior to 2000. The factor became larger after 2000 reaching a maximum of 23.3 basis points in January 2001 but again narrowed after 2004 as credit spreads narrowed. It is notable that the larger impact of credit risk occurred in a period of steeper credit spread slopes in the corporate bond market, consistent with a fear of larger future LIBOR spreads. This highlights the importance of incorporating the term structure of corporate bonds in modeling swap spreads, since the slope of the corporate curve has information on the credit component in long-term swap rates that cannot be deduced from a 'short' credit spread alone.

Our model not only estimates the impact of the swap factor through time but also allows us to see its term structure implications. As shown in Table 5, the term premium arising due to the swap factor is virtually constant and negative by around 8.0 basis points. Johannes and Sundaresan (2007) argue that the posting of collateral by both parties to the swap contract has the net effect of increasing swap yields. The fact that we view the swap contract as free of counterparty risk is consistent with collateral being posted. However, when controlling for the convenience yield in Treasuries and credit risk in LIBOR, we do not find support for their hypothesis since the swap factor has a constant and on average negative effect on swap rates across maturity.

Finally, our decomposition of swap spreads allows us to measure the distance from the riskless rate to the Treasury rate and to the swap rate. Figs. 9 and 10 show, respectively, the 2-year and 10-year spreads between the Treasury par rate and the riskless par rate, between the swap rate and the riskless par rate, and between the AAA par bond yield and the riskless rate. We see that throughout the sample, the 2-year AAA rate is a close proxy for the riskless rate, supporting a common practice of measuring true credit spreads for lower credits by subtracting the AAA spread. Consistent with the possibility of a downgrade, longer-term AAA spreads cannot be viewed as a riskless rate. The swap rate is on average not far from the riskless rate at either maturity, but it deviates particularly in the period 2002–2003. The Treasury rates are consistently far from the estimated riskless rates.

5.4. Liquidity of corporate bonds

In our model we have assumed that there is a convenience yield to holding Treasuries but that there is no illiquidity premium to holding corporate bonds. To test how the results are affected if such an illiquidity

Table 5
The average effect in basis points of the swap-specific factor, Treasury convenience yield, and LIBOR credit risk on swap rates across maturities

Maturity	2	3	5	7	10
Average swap effect	−7.9	−7.9	−8.0	−8.1	−8.3
Average Treasury effect	50.0	51.0	52.9	54.5	56.8
Average credit risk effect	5.5	6.6	8.3	9.5	10.8

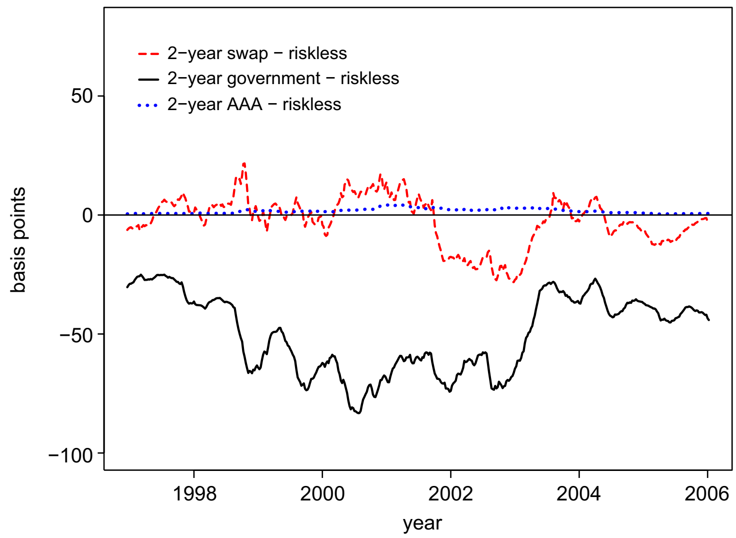

Fig. 9. The distance from the estimated 2-year riskless yield to the corresponding estimated Treasury yield, AAA yield, and swap rate.

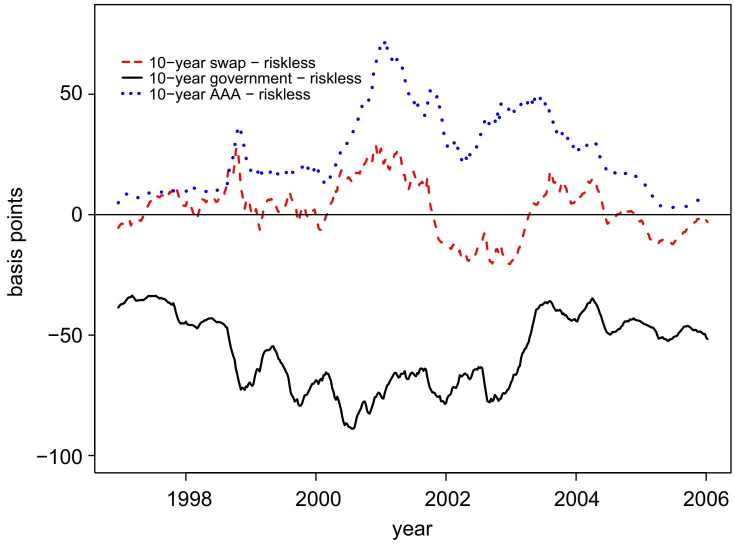

Fig. 10. The distance from the estimated 10-year riskless yield to the corresponding estimated Treasury yield, AAA yield, and swap rate.

premium is present, we conduct the following two experiments. First, we add a constant illiquidity premium of 20 basis points to all corporate bond yields and re-estimate the model. The average decomposition of swap spreads in this experiment is given in Table 6. Second, we increase all corporate–Treasury spreads by 20% and re-estimate the model, and again the results are in Table 6. These experiments show that an illiquidity factor in corporate yields is unlikely to affect our estimate of the credit risk component in swap spreads. In both cases, the credit spread component of swap spreads changes by a maximum of 1.3 basis points—hence a smaller effect. Also, we see that the illiquidity factor is more or less absorbed in both the convenience yield and the swap factor, resulting in a higher convenience yield and a lower swap factor. It is therefore conceivable that part of the convenience yield is due to an illiquidity premium in corporate bonds. However, there are several reasons why we do not think that this is a large effect. First, our swap factor, which measures deviations from the homogeneous credit risk quality assumption between LIBOR and AA, has an average over the entire sample period of −8.0 basis points and an average of −2.9 basis points excluding the extreme period between October 2001 and October 2003. If the swap factor has a sizeable corporate bond illiquidity component, it would imply that the swap curve has a sizeable positive liquidity component. Second, the AA index in our sample is based on very liquid bonds. The liquidity spread for 1- to 7-year AA-rated bonds is estimated in Chen, Lesmond, and Wei (2007) to be 9.63 basis points. However, the bonds in our AA sample are all more liquid according to the zero-return measure than the average of AA bonds employed in their paper. This measure records the percentage of trading days on which the bond price does not change (hence has zero return) indicating that there has been no trading in the bond. In their paper, the average proportion of zero returns is 4.10% whereas in our AA sample it is 0.02%. All in all, this further suggests that if there is an illiquidity premium in the corporate bonds in our data set, it is in fact small and unlikely to change our conclusion that the riskless rate and the Treasury rate are separated by a large spread and that this is the major component of the swap spread.

6. Conclusion

We analyze a six-factor model for Treasury bonds, corporate bonds, and swap rates and decompose the swap spreads for each time and maturity into three components: one arising from a convenience yield to holding Treasuries, one explained by the credit risk element of the underlying LIBOR rate, and one factor specific to the swap market. We have seen that in the later part of our sample, the swap-specific factor is related to hedging activity in the mortgage-backed security market even if other events in the fixed income markets prevent us from pointing to this as the only cause.

Table 6
The effect of a possible liquidity component in corporate bonds. Panel A shows the average effect in basis points of the swap-specific factor, Treasury convenience yield, and LIBOR credit risk on swap rates across maturities. Panel B shows the average effect when 20 basis points are added to all corporate yields and the model is re-estimated. Panel C shows the average effect when corporate–Treasury yield spreads are increased by 20% and the model is re-estimated.

Maturity	2	3	5	7	10
Panel A: base case					
Average swap effect	−7.9	−7.9	−8.0	−8.1	−8.3
Average Treasury effect	50.0	51.0	52.9	54.5	56.8
Average credit risk effect	5.5	6.6	8.3	9.5	10.8
Panel B: adding 20 basis points to all corporate– Treasury yield spreads					
Average swap effect	−27.8	−27.7	−27.4	−27.1	−26.9
Average Treasury effect	70.4	71.2	72.8	74.3	76.2
Average credit risk effect	5.0	6.1	7.9	9.1	10.5
Panel C: scaling up all corporate– Treasury yield spreads by 20%					
Average swap effect	−17.9	−17.9	−17.9	−18.1	−18.3
Average Treasury effect	59.6	60.5	62.1	63.5	65.4
Average credit risk effect	5.9	7.2	9.2	10.6	12.1

The credit risk in LIBOR contributes to swap spreads and it is on average increasing with the maturity of the swap. We find that information in the corporate bond market is better at capturing the credit risk in swap rates than the commonly used proxy LIBOR-GC repo. The Treasury factor accounts for most of the swap spread in the entire sample. The average effect of the swap factor is virtually constant as a function of maturity but is time-varying. As a consequence of these observations, the riskless rate is better proxied by the swap rate than the Treasury rate for all maturities.

Appendix A. A result on univariate affine processes

For the univariate 'translated CIR' affine process

$$dX_t = k(X_t - \theta)\,dt + \sqrt{\alpha + \beta X_t}\,dW, \tag{A.1}$$

and with a payment that is an exponential function of the state variables, we know from Duffie, Pan, and Singleton (2000) that there exist A and B such that

$$E_t(e^{-\int_t^T c_1 X_u\,du} e^{c_2 X_T}) = e^{A(t,T)+B(t,T)X_t}. \tag{A.2}$$

We find the solution with explicitly calculated coefficients in Christensen (2002) and list them here for convenience:

$$\begin{aligned}
B(t,T) &= \frac{-2c_1(e^{\gamma(T-t)}-1) + c_2 e^{\gamma(T-t)}(\gamma+k) + c_2(\gamma-k)}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)}-1)},\\
A(t,T) &= \frac{-2k\theta}{\beta}\ln\left(\frac{2\gamma e^{(1/2)(\gamma-k)(T-t)}}{2\gamma + (\gamma-k-c_2\beta)(e^{\gamma(T-t)}-1)}\right)\\
&\quad + \frac{1}{2}\alpha\frac{(2c_1 + c_2(\gamma-k))^2}{(\gamma+k+c_2\beta)^2}(T-t)\\
&\quad + \frac{2\alpha k}{\beta^2}\ln\left(\frac{2\gamma + (\gamma-k-c_2\beta)(e^{\gamma(T-t)}-1)}{2\gamma}\right)\\
&\quad - \frac{2\alpha}{\beta}\frac{(e^{\gamma(T-t)}-1)(c_1 - kc_2 - \frac{1}{2}\beta c_2^2)}{2\gamma + (\gamma-k-c_2\beta)(e^{\gamma(T-t)}-1)},\\
\gamma &= \sqrt{k^2 + 2\beta c_1} \quad \text{for } c_1 > -\frac{k^2}{2\beta}.
\end{aligned}$$

Appendix B. Pricing formulas

B.1. Riskless bonds

The riskless rate is given in Eq. (13) as

$$r = X_1 + X_2 + X_5.$$

As noted in Section 4, in order to have identification of the parameters we set the means of the processes to zero and add a constant,

$$r = (a + X_1 + X_2) + (e + X_5).$$

Prices of riskless bonds are

$$\begin{aligned}
P(t,T) &= E_t\left(\exp\left(-\int_t^T r(u)\,\mathrm{d}u\right)\right)\\
&= \mathrm{e}^{-(a+e)(T-t)}\mathrm{e}^{-\int_t^T X_{1u}\,\mathrm{d}u}\mathrm{e}^{-\int_t^T X_{2u}\,\mathrm{d}u}\mathrm{e}^{-\int_t^T X_{5u}\,\mathrm{d}u}\\
&= \mathrm{e}^{-(a+e)(T-t)}\mathrm{e}^{A^1(T-t)+B^1(T-t)X_{1t}}\mathrm{e}^{A^2(T-t)+B^2(T-t)X_{2t}}\mathrm{e}^{A^5(T-t)+B^5(T-t)X_{1t}}\\
&= \mathrm{e}^{A(T-t)+\mathbf{B(T-t)}'X_t},
\end{aligned}$$

where we have used the independence of the processes and a special case of the result in Appendix A, with A and $\mathbf{B}$ given as

$$\begin{aligned}
&A(T-t) = -(a+e)(T-t) + A^1(T-t) + A^2(T-t) + A^5(T-t),\\
&\mathbf{B(T-t)}' = (B^1(T-t), B^2(T-t), 0, 0, B^5(T-t), 0).
\end{aligned}$$

B.2. Government bonds

In Eq. (12) the government rate is given as

$$r^{\mathrm{g}} = a + X_1 + X_2,$$

and the prices of zero-coupon government bonds are

$$P^{\mathrm{g}}(t,T) = \mathrm{e}^{A^{\mathrm{g}}(T-t)+\mathbf{B}^{\mathrm{g}}\mathbf{(T-t)}'X_t},$$

where $A^{\mathrm{g}}(T-t) = -a(T-t) + A^1(T-t) + A^2(T-t)$ and $B^{\mathrm{g}}(T-t)' = (B^1(T-t), B^2(T-t), 0, 0, 0, 0)$ are derived exactly as in the riskless bond case. In the empirical work we use par rates, and the T-year par rate $y^{\mathrm{par}}(t,T)$ is easily expressed as

$$y^{\mathrm{par}}(t,T) = 2\frac{1 - P^{\mathrm{g}}(t,T)}{\sum_{i=1}^{2T} P^{\mathrm{g}}\left(t, t+\frac{i}{2}\right)}.$$

B.3. Corporate bonds

In the pricing of corporate bonds we choose to work with a generator matrix excluding default states,

$$\tilde{A}_X(s) = \tilde{\Lambda}^v \mu(X_s) = \begin{pmatrix} -\lambda_1 - v_1 & \lambda_{12} & \cdots & \lambda_{1,K-1} \\ \lambda_{21} & -\lambda_2 - v_2 & \cdots & \lambda_{2,K-1} \\ \vdots & & \ddots & \vdots \\ \lambda_{K-1,1} & \cdots & \cdots & -\lambda_{K-1,K-1} - v_{K-1} \end{pmatrix} \mu(X_s),$$

for notational reasons. We can decompose $\tilde{\Lambda}^v$ into $\tilde{\Lambda}^v = \tilde{B}\tilde{D}\tilde{B}^{-1}$, where $\tilde{D}$ is a diagonal matrix with the eigenvalues of $\tilde{\Lambda}^v$ in the diagonal and $\tilde{B}$ is a $K-1 \times K-1$ matrix with columns given by the $K-1$ eigenvectors of $\tilde{\Lambda}^v$. Defining $[\tilde{B}^{-1}]_{j,K} = -\sum_{k=1}^{K-1}[\tilde{B}]_{jk}$, the price of a corporate bond with rating i can be written as

$$v^i(t,T) = \sum_{j=1}^{K-1} -[\tilde{B}]_{ij}[\tilde{B}^{-1}]_{j,K} E_t(\mathrm{e}^{\int_t^T \tilde{D}_{jj}\mu(X_u) - r(X_u)\,\mathrm{d}u}) \tag{B.1}$$

according to Lando (1998), so we have $c_{ij} = -[\tilde{B}]_{ij}[\tilde{B}^{-1}]_{j,K}$ and $d_j = \tilde{D}_{jj}$ in Eq. (5). From the specification in Eqs. (13) and (14) we have that $r = a + X_1 + X_2 + (e + X_5)$ and $\mu = b + X_3 + X_4 + c(X_1 + X_2)$ so we can solve the conditional expectation

$$\begin{aligned}
&E_t(\mathrm{e}^{\int_t^T \tilde{D}_{jj}\mu(X_u) - r(X_u)\,\mathrm{d}u}) \\
&= \mathrm{e}^{-(T-t)(a+e-\tilde{D}_{jj}b)} E_t(\mathrm{e}^{-\int_t^T ((1-c\tilde{D}_{jj})X_{1u} + (1-c\tilde{D}_{jj})X_{2u} + (-\tilde{D}_{jj})X_{3u} + (-\tilde{D}_{jj})X_{4u} + X_{5u}\,\mathrm{d}u)}) \\
&= \mathrm{e}^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{2} [E_t(\mathrm{e}^{-\int_t^T (1-c\tilde{D}_{jj})X_{iu}\,\mathrm{d}u})] \prod_{i=3}^{4} [E_t(\mathrm{e}^{-\int_t^T (-\tilde{D}_{jj})X_{iu}\,\mathrm{d}u})] E_t(\mathrm{e}^{-\int_t^T (X_{5u}\,\mathrm{d}u})) \\
&= \mathrm{e}^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{5} [\mathrm{e}^{A_i^j(T-t) + B_i^j(T-t)X_{it}}] = \mathrm{e}^{A^j(T-t) + B^j(T-t)' X_t},
\end{aligned}$$

where

$$\begin{aligned}
A^j(T-t) &= -(T-t)(a + e - \tilde{D}_{jj}b) + \sum_{i=1}^{5}[A_i^j(T-t)], \\
B^j(T-t)' &= (B_1^j(T-t), \ldots, B_5^j(T-t), 0).
\end{aligned}$$

B.4. Swap rates

To price interest rate swaps we value the floating-rate payments and fixed-rate payments separately. In the following we assume that the floating rate is paid quarterly while the fixed rate is paid semiannually. With n being the maturity of the swap in quarters of a year, the present value of the floating-rate payments in the swap is

$$E_t^Q\left[\sum_{i=1}^{n} \mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\left(\frac{a(t_{i-1},t_i)}{360} L(t_{i-1},t_i)\right)\right] = E_t^Q\left[\sum_{i=1}^{n} \mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\left(\frac{1}{v^{\mathrm{LIB}}(t_{i-1},t_i)} - 1\right)\right], \tag{B.2}$$

where $a(t_{i-1}, t_i)$ is the actual number of days between time t_{i-1} and time t_i. The present value for the fixed-rate payments is

$$\frac{F(t,T)}{2} \sum_{i=1}^{n/2} P(t, t_{2i}). \tag{B.3}$$

In evaluating the present value of the floating-rate payments we follow the idea outlined in Duffie and Liu (2001). The ith floating-rate payment can be rewritten as

$$E_t^Q\left[\mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\left(\frac{1}{v^{\mathrm{LIB}}(t_{i-1},t_i)} - 1\right)\right] = E_t^Q\left[\mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\left(\frac{1}{v^{\mathrm{LIB}}(t_{i-1},t_i)}\right)\right] - P(t,t_i). \tag{B.4}$$

Assumption (7) and (8) give

$$
\begin{aligned}
v^{\mathrm{LIB}}(t_{i-1},t_i) &= E^Q_{t_{i-1}}(\mathrm{e}^{-\int_{t_{i-1}}^{t_i} v_2\mu(X_u)+r_u+d+X_6\,\mathrm{d}u}) \\
&= \mathrm{e}^{-0.25(a+d+e+v_2b)} \prod_{j=\{1,2,5,6\}} E^Q_{t_{i-1}}(\mathrm{e}^{-\int_{t_{i-1}}^{t_i} X_{ju}\,\mathrm{d}u}) \prod_{j=\{3,4\}} E^Q_{t_{i-1}}(\mathrm{e}^{-\int_{t_{i-1}}^{t_i} v_2 c X_{ju}\,\mathrm{d}u}) \\
&= \mathrm{e}^{-0.25(a+d+e+v_2b)} \prod_{j=1}^{6} \mathrm{e}^{\overline{A}_j(0.25)+\overline{B}_j(0.25)X_{jt_{i-1}}} = \mathrm{e}^{\overline{A}(0.25)+\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}}, \qquad \text{(B.5)}
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{A}(0.25) &= -0.25(a+d+e+v_2b) + \sum_{j=1}^{6} \overline{A}_j(0.25), \\
\overline{\mathbf{B}}(0.25)' &= (\overline{B}_1(0.25),\ldots,\overline{B}_6(0.25)).
\end{aligned}
$$

By the law of iterated expectations the expectation in the ith floating-rate payment given in Eq. (B.4) is

$$
\begin{aligned}
E^Q_t\left[\mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\frac{1}{v^{\mathrm{LIB}}(t_{i-1},t_i)}\right] &= E^Q_t\left[\mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\mathrm{e}^{-\overline{A}(0.25)-\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}}\right] \\
&= \mathrm{e}^{-\overline{A}(0.25)} E^Q_t\left[\mathrm{e}^{-\int_t^{t_{i-1}} r_u\,\mathrm{d}u}\mathrm{e}^{-\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}} E^Q_{t_{i-1}}\left[\mathrm{e}^{-\int_{t_{i-1}}^{t_i} r_u\,\mathrm{d}u}\right]\right],
\end{aligned}
$$

and since

$$
E^Q_{t_{i-1}}[\mathrm{e}^{-\int_{t_{i-1}}^{t_i} r_u\,\mathrm{d}u}] = \mathrm{e}^{-0.25(a+e)} \prod_{j=\{1,2,5\}} E^Q_{t_{i-1}}(\mathrm{e}^{-\int_{t_{i-1}}^{t_i} X_{ju}\,\mathrm{d}u}) = \mathrm{e}^{\overline{\overline{A}}(0.25)+\overline{\overline{\mathbf{B}}}(0.25)'X_{t_{i-1}}},
$$

where

$$
\begin{aligned}
\overline{\overline{A}}(0.25) &= -0.25(a+e) + \sum_{j=\{1,2,5\}} \overline{A}_j(0.25), \\
\overline{\overline{\mathbf{B}}}(0.25) &= (\overline{B}_1(0.25),\overline{B}_2(0.25),0,0,\overline{B}_5(0.25),0),
\end{aligned}
$$

we have

$$
\begin{aligned}
&E^Q_t\left[\mathrm{e}^{-\int_t^{t_i} r_u\,\mathrm{d}u}\frac{1}{v^{\mathrm{LIB}}(t_{i-1},t_i)}\right] \\
&\quad = \mathrm{e}^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)} E^Q_t\left[\mathrm{e}^{-\int_t^{t_{i-1}} r_u\,\mathrm{d}u}\mathrm{e}^{(\overline{\overline{\mathbf{B}}}(0.25)-\overline{\mathbf{B}}(0.25))'X_{t_{i-1}}}\right] \\
&\quad = \mathrm{e}^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} \\
&\qquad \times E^Q_t\left[\mathrm{e}^{-\int_t^{t_{i-1}} X_{1u}+X_{2u}+X_{5u}\,\mathrm{d}u}\mathrm{e}^{-\overline{B}_3(0.25)X_{3t_{i-1}}-\overline{B}_4(0.25)X_{4t_{i-1}}-\overline{B}_6(0.25)X_{6t_{i-1}}}\right] \\
&\quad = \mathrm{e}^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} \prod_{j=\{1,2,5\}} E^Q_t\left[\mathrm{e}^{-\int_t^{t_{i-1}} X_{ju}\,\mathrm{d}u}\right] \prod_{j=\{3,4,6\}} E^Q_t[\mathrm{e}^{-\overline{B}_j(0.25)X_{jt_{i-1}}}] \\
&\quad = \mathrm{e}^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} \prod_{j=1}^{6} \mathrm{e}^{\overline{\overline{\overline{A}}}_j(t_{i-1}-t)+\overline{\overline{\overline{B}}}_j(t_{i-1}-t)X_{jt}} \\
&\quad = \mathrm{e}^{A^{\mathrm{s}}(t_{i-1}-t)+\mathbf{B}^{\mathrm{s}}(t_{i-1}-t)'X_t},
\end{aligned}
$$

where

$$A^{s}(t_{i-1}-t)=0.25v_2b-\sum_{j=3}^{4}\overline{A}_j(0.25)-(t_{i-1}-t)(a+e)+\sum_{j=1}^{6}\overline{\overline{\overline{A}}}_j(t_{i-1}-t),$$

$$\mathbf{B}^{s}(t_{i-1}-t)'=(\overline{\overline{\overline{B}}}_1(t_{i-1}-t),\ldots,\overline{\overline{\overline{B}}}_6(t_{i-1}-t)).$$

Inserting this in formula (B.2) for the floating-rate payments,

$$\sum_{i=1}^{n}(\mathrm{e}^{A^{s}(t_{i-1}-t)+\mathbf{B}^{s}(t_{i-1}-t)'X_t}-P(t,t_i)),$$

and equating the present value of the fixed- and floating-rate payments we get the swap rate

$$F(t,T)=\frac{2\sum_{i=1}^{n}(\mathrm{e}^{A^{s}(t_{i-1}-t)+\mathbf{B}^{s}(t_{i-1}-t)'X_t}-P(t,t_i))}{\sum_{i=1}^{n/2}P(t,t_{2i})}.$$

When calculating the current 3-month LIBOR rate we allow for downgrades, so it is calculated as

$$L(t,t+0.25)=\frac{360}{a(t,t+0.25)}\left[\frac{1}{v^{\mathrm{LIB}}(t,t+0.25)}-1\right],$$

where

$$\begin{aligned}v^{\mathrm{LIB}}(t,t+0.25)&=\sum_{j=1}^{K-1}-[B]_{ij}[B^{-1}]_{j,K}E_t(\mathrm{e}^{\int_t^T\tilde{D}_{jj}\mu(X_u)-(r(X_u)+d+X_{6u})\,\mathrm{d}u})\\&=\left(\sum_{j=1}^{K-1}-[B]_{ij}[B^{-1}]_{j,K}E_t(\mathrm{e}^{\int_t^T\tilde{D}_{jj}\mu(X_u)-r(X_u)\,\mathrm{d}u})\right)E_t(\mathrm{e}^{-\int_t^T d+X_{6u}\,\mathrm{d}u})\\&=v^{\mathrm{AA}}(t,t+0.25)E_t(\mathrm{e}^{-\int_t^T d+X_{6u}\,\mathrm{d}u}).\end{aligned}$$

Appendix C. The Kalman filter

The Kalman filter is an algorithm that estimates the unobserved state variables and calculates the likelihood function in the state space model

$$y_t=A_t+B_tX_t+\varepsilon_t,\quad \varepsilon_t\sim\mathrm{N}(0,H_t),\tag{C.1}$$

$$X_t=C_t+D_tX_{t-1}+\eta_t,\quad \eta_t\sim\mathrm{N}(0,Q_t).$$

The algorithm consists of a sequence of prediction and update steps. Let

$$\hat{F}_t=\sigma(y_1,y_2,\ldots,y_{t-1},y_t)$$

denote the information set available at time t. We follow Harvey (1990, Chapter 3), where all derivations can be found. At time $t-1$, we have estimated the state variable $\hat{X}_{t-1}$ and the mean square error matrix $\hat{P}_{t-1}=E((X_{t-1}-\hat{X}_{t-1})(X_{t-1}-\hat{X}_{t-1})'|\hat{F}_{t-1})$. The state variable in the next period, X_t, is predicted in the *prediction step* by

$$\hat{X}_{t|t-1}=E(X_t|\hat{F}_{t-1})=C_t+D_t\hat{X}_{t-1}\tag{C.2}$$

and the mean square error matrix by

$$\hat{P}_{t|t-1}=E((X_t-\hat{X}_{t|t-1})(X_t-\hat{X}_{t|t-1})'|\hat{F}_{t-1})=D_t\hat{P}_{t-1}D_t'+Q_t.\tag{C.3}$$

Next, the additional information contained in y_t is used to obtain a more precise estimator of X_t in the *update step*

$$\hat{X}_t=E(X_t|\hat{F}_t)=\hat{X}_{t|t-1}+\hat{P}_{t|t-1}B_t'F_t^{-1}v_t\tag{C.4}$$

with mean square error matrix

$$\hat{P}_t = \hat{P}_{t|t-1} - \hat{P}_{t|t-1}B_t'F_t^{-1}B_t\hat{P}_{t|t-1}, \tag{C.5}$$

where

$$v_t = y_t - E(y_t|\hat{F}_{t-1}) = y_t - (A_t + B_t\hat{X}_{t|t-1}),$$
$$F_t = Cov(v_t) = B_t\hat{P}_{t|t-1}B_t' + H_t.$$

The estimator of X_t in (C.4) is called the *filtered* estimator. We need initial values of X_0 and P_0 to start the recursions. If the state vector X_t is stationary, we can use the unconditional mean and covariance matrix of X_t. Finally, the log-likelihood function is

$$\log L(y_1,\ldots,y_t;\psi) = \sum_{k=1}^{t} -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log|F_k| - \frac{1}{2}v_k'F_k^{-1}v_k, \tag{C.6}$$

where ψ contains all the parameters of the state space model. Although not written explicitly earlier, A, B, D, and Q_t are functions of ψ. To estimate the parameters ψ, the log-likelihood function is maximized over ψ.

C.1. Missing observations

A considerable advantage of the state space approach is the ease with which missing observations can be dealt. Suppose that at time t some of the elements of the N-observation vector y_t are missing. We let y_t^* be the L-vector of values actually observed where $L<N$ and follow Harvey (1990, Section 3.4.7). We can write $y_t^* = W_t y_t$, where W_t is a known $L \times N$ matrix whose rows are a subset of the rows of I_N. At time points where not all elements of y_t are available, Eq. (C.1) is replaced by the equation

$$y_t^* = A_t^* + B_t^* X_t + \varepsilon_t^*, \quad \varepsilon_t^* \sim \mathrm{N}(0, H_t^*),$$

where

$$A_t^* = W_t A_t, \quad B_t^* = W_t B_t, \quad H_t^* = W_t H_t W_t'.$$

Otherwise the Kalman filter is applied in the same way except that N in the log-likelihood function (C.6) is time-varying.

C.2. The extended Kalman filter

In this section we state the adjustments necessary to the Kalman filter if the measurement equation is nonlinear in the state variables. Again, we follow Harvey (1990, Chapter 3.7.2).

Consider the state space model

$$y_t = f_t(X_t) + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{N}(0, H_t),$$
$$X_t = C_t + D_t X_{t-1} + \eta_t, \quad \eta_t \sim \mathrm{N}(0, Q_t),$$

where the function $f : \mathbf{R}^m \to \mathbf{R}^N$ is nonlinear. If f is sufficiently smooth, the function can be expanded around the forecast of X_t, $\hat{X}_{t|t-1}$,

$$f_t(X_t) \simeq f_t(\hat{X}_{t|t-1}) + \hat{B}_t(X_t - \hat{X}_{t|t-1}),$$

where

$$\hat{B}_t = \frac{\partial f_t(x)}{\partial x}\Big|_{x=\hat{X}_{t|t-1}}.$$

This leads to an approximation of the original nonlinear filter by

$$y_t = \hat{A}_t + \hat{B}_t X_t + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{N}(0, H_t),$$
$$X_t = C_t + D_t X_{t-1} + \eta_t, \quad \eta_t \sim \mathrm{N}(0, Q_t),$$

where

$$\hat{A}_t = f_t(\hat{X}_{t|t-1}) - \hat{B}_t \hat{X}_{t|t-1}.$$

The Kalman filter in the previous section is applied to the linearized model with one modification: the variable v_t is calculated as

$$v_t = y_t - f_t(\hat{X}_{t|t-1}).$$

Appendix D. Estimation details

D.1. Optimization

The optimization of the likelihood function is complicated for two reasons. First, in the Kalman filter recursions it might happen that $\hat{X}_t < -\alpha/\beta$ in which case we set $\hat{X}_t = -\alpha/\beta$ to ensure that $Var(X_t|X_{t-1})$ remains positive definite. In this case we follow Duffee and Stanton (2001) and set the log-likelihood function to a large negative number. Second, the likelihood function has a large number of local maxima as is common in these types of models. We employ the usual procedure of repeatedly generating a random vector of starting values and maximize the log-likelihood function. This was done 100 times using the Nelder-Mead maximization algorithm and the largest of the 100 resulting values was chosen. For the 10 largest values we compared the parameter values and they were generally in the same range. More importantly in regard to the conclusions in our paper, the filtered processes were very close to each other across the ten values.

D.2. Standard errors

Because the log-likelihood function is misspecified for non-Gaussian models, a robust estimate of the variance-covariance matrix can be found using White (1982) as

$$\hat{\Sigma}_1 = \frac{1}{T}[\hat{A}\hat{B}^{-1}\hat{A}]^{-1}, \tag{D.1}$$

where

$$\hat{A} = -\frac{1}{T}\sum_{i=1}^{T}\frac{\partial^2 \log l_t(\hat{\theta})}{\partial\theta\partial\theta'},$$

$$\hat{B} = \frac{1}{T}\sum_{i=1}^{T}\frac{\partial \log l_t(\hat{\theta})}{\partial\theta}\frac{\partial \log l_t(\hat{\theta})'}{\partial\theta}.$$

In order to minimize the concern of numerical instability in the calculation of second derivatives, we estimate "smoothed" versions of $\hat{A}$ and $\hat{B}$ which are calculated as follows. The $\Delta\theta_1$ and $\Delta\theta_2$ vectors leading to the most stable calculation of first and second derivatives are found. $\hat{A}$ is found by calculating $\hat{A}_i$ using $(0.8+0.02i)\Delta\theta_1$, $i = 1,\ldots,20$, and letting $\hat{A} = E(\hat{A}_i)$. $\hat{B}$ is found by calculating $\hat{B}_i$ using $(0.8+0.02i)\Delta\theta_1$ and $(0.8+0.02i)\Delta\theta_2$, $i = 1,\ldots,20$, and letting $\hat{B} = E(\hat{B}_i)$. Standard errors using the smoothed estimates and formula (D.1) are reported in the first row after parameter estimates. In the second row after parameter estimates we report standard errors using the theoretically less but numerically more robust estimator of the variance–covariance matrix,

$$\hat{\Sigma}_2 = \frac{1}{T}\left[\frac{1}{T}\sum_{i=1}^{T}\frac{\partial \log l_t(\hat{\theta})}{\partial\theta}\frac{\partial \log l_t(\hat{\theta})'}{\partial\theta}\right]^{-1} = [T\hat{B}]^{-1},$$

where the smoothed estimate of $\hat{B}$ is used.

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